Three Traditions of Self-Similarity in Fourteenth and Fifteenth Century Islamic Geometric Ornament

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Abstract

As early as the ninth century Muslim artists began incorporating multiple-level designs into their ornament. In time, this fascination found expression in each of the three principle areas of Islamic two-dimensional artistic expression: calligraphy, the floral idiom, and geometric pattern. In both the western regions of Morocco and Andalusia, and the eastern regions of Persia, Khurasan and Transoxiana, during the fourteenth and fifteenth centuries respectively, previously established systematic methods of two-dimensional geometric pattern construction were used in the development of three distinct traditions of self-similar geometric design. These innovations resulted in the last great advancement in the long history of Islamic geometric pattern making. Furthermore, these patterns are very likely the first, and among the most engaging, examples of complex overtly self-similar art made by man.

The traditional techniques used in the creation of Islamic self-similar patterns, overviewed herein, are easily learned and open to further creative development. In addition to new and original two-dimensional periodic designs (as per historic examples), further areas of stylistically Islamic self-similar geometric pattern innovation can include their application to aperiodic tilings, as well as non-Euclidean hyperbolic and spherical surface coverage.

Historical Background

The Islamic interest in ever more complex and varied ornament eventually led to the development of several remarkable traditions of multiple-level design. This form of ornament augments a bold primary motif by incorporating smaller scale secondary elements into the background areas. The antecedents for this type of Islamic design can be traced back to the ninth century window grilles in the Ibn Tulun Mosque in Cairo [1]. While these early geometric examples are rare, in time the Islamic fascination with multiple-level ornament found wide favor within each of the three principle modes of Islamic artistic expression: floral design, calligraphy, and geometric pattern. The tradition of multiple-level floral designs reached maturity during the fourteenth century, and while notable examples can be found from throughout the Islamic world, the Timurid emphasis on this tradition resulted in designs of consistently the highest quality. An exceedingly fine example, undertaken in relief carved marble, can be found on the fifteenth century tombstone of Ghiyathuddin Mansur in the madrasa of Sultan Husayn Mirza in Herat [**Figure 1**]. The background to the primary floral scrollwork provides the area for the further inclusion of a smaller, secondary floral device that is of near identical style. The Safavids also favored this form of floral ornament, and while influenced by the earlier Timurid style, their work is stylistically distinct, and

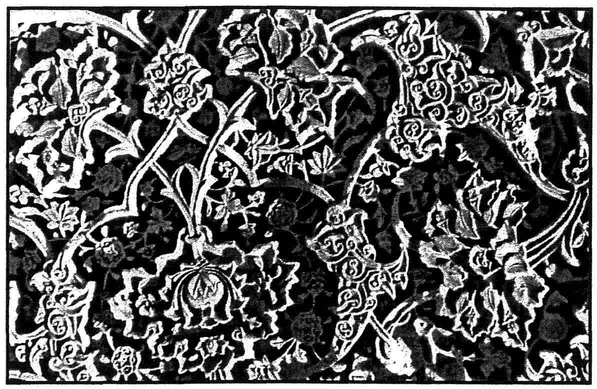

Figure1: *Multiple-level floral design from the tombstone of Ghiyathuddin at the madrasa of Sultan Husayn Mirza, Herat.* Photo: courtesy of Bernard O'Kane.

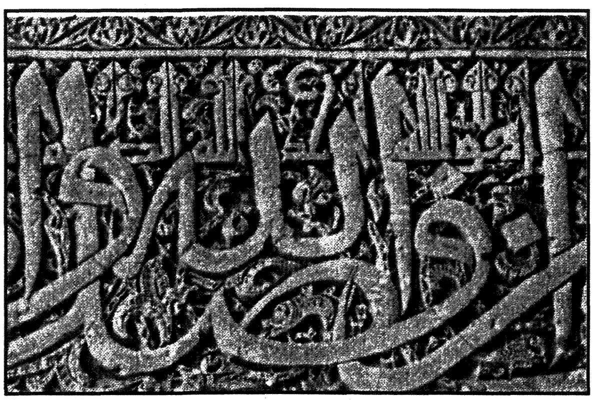

Figure 2: *Multiple-level Thuluth and Kufic calligraphic ornament from the Masjid-i Jami, Varamin.*

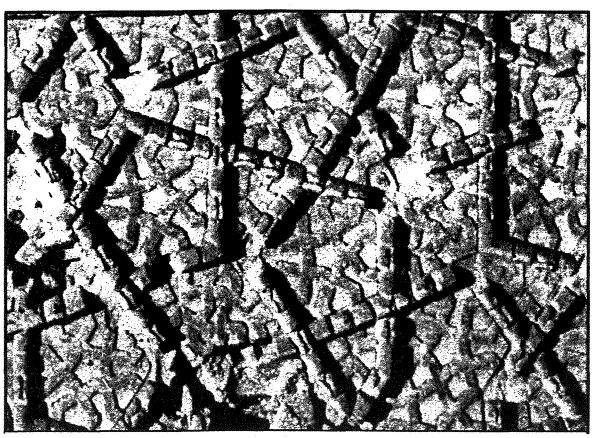

Figure 3: *Multiple-level geometric design from the Gunbad-i Kabud, Maragha.*

includes a variety of stylistic variations. Most noteworthy among these is the floral technique known as *yezdi bendi* wherein the background of the primary floral scroll contains a thinner parallel secondary scroll with proportionally reduced flower and leaf forms [2].

Within the Islamic calligraphic tradition, multiple-level design was primarily used as a form of architectural ornament, where far greater stylistic variation was practiced than in the book arts. Multiple-level calligraphic ornament incorporates a smaller secondary calligraphic element into the background of the primary text. This calligraphic technique most often involved the placing of a smaller scale band of angular *Kufi* calligraphy [3] behind the upper portion of ascending letters such as the *alif* [4] in cursive scripts such as *Thuluth* [5]. Many epigraphic examples of this form of calligraphic ornament were produced, particularly in Persia. A lovely late Ilkhanid example can be seen in the carved stucco ornament of the Masjid-i Jami in Varamin [**Figure 2**]. The many historic examples of floral and calligraphic multiple-level designs are unquestioningly beautiful and imbued with creative ingenuity. However, in themselves, these multiple-level designs do not represent a development of significant importance in the overall history of Islamic art. For this, we must look to the inspired use of geometry within the Islamic ornamental arts.

The Gunbad-i Kabud tomb tower in Maragha, Azerbaijan (1196-97), is among the most interesting of the early surviving examples of Islamic multiple-level geometric ornament [**Figure 3**]. As with both floral and calligraphic multiple-level designs, the background of the primary geometric design is further elaborated with the addition of a secondary geometric motif. While not self-similar, the pattern used in this late Seljuq building is noteworthy for its level of multiple-level sophistication at such an early date. The primary geometric pattern that covers the exterior façade is composed of a series of convex and concave polygons that are typical to the *obtuse* family of 5-fold geometric patterns [6]. This variety of 5-fold geometric pattern was well established in the eastern Islamic regions by the time the Gunbad-I Kabud tomb tower was constructed. The unusual feature in this example is that each of the shapes that make up the pattern has a smaller scale, secondary geometric motif applied within it. Patterns

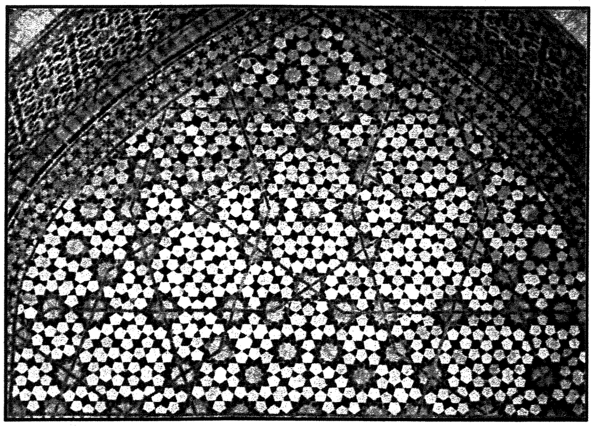

Figure 4: *Self-Similar Type A design from the Darb-i Imam, Isfahan.* Photo: courtesy of Kendall Dudley.

such as this can be referred to as *additive patterns*, and were used to a limited extent by the Seljuqs, and more widely by the later Ilkhanids. The technique of making standard patterns more complex by a simple additive process is in marked contrast to the more demanding technique of constructing the fully mature self-similar patterns that were to follow. Yet this particular multiple-level additive pattern closely approximates the aesthetics of later Self-Similar Type A examples, and is a clear indication of a trend toward ever-greater multiple-level complexity. As such, it can be regarded as formative in the process toward self-similarity. Other formative examples from the eastern regions include the minaret of the Yakutiye Madrasa in Erzurum in Turkey (1310), and on the Dha'l Kifl minaret in al-Kifl, Iraq (1316). Each of these three formative examples were used on the exterior surfaces of their respective buildings and can be seen from both near and far; allowing for the dynamics of scale to provide travelers with a progressive appreciation of the primary design from a relatively great distance, and the secondary elements upon closer proximity.

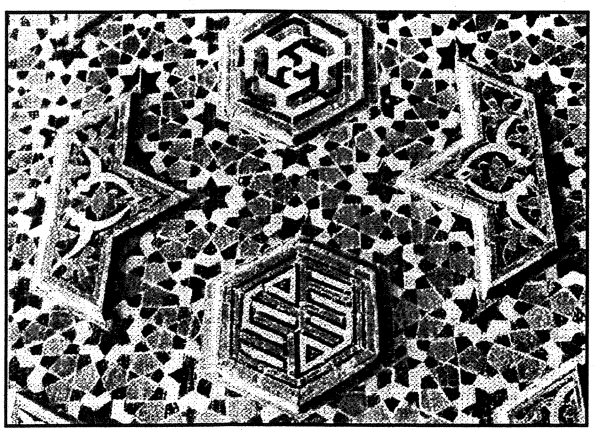

Figure 5: *Self-Similar Type B design from the Masjid-i Jami, Varzana.* Photo: courtesy of Antony Hutt and Leonard Harrow.

Islamic self-similar geometric patterns developed along two separate historic paths: that of the eastern regions of Persia, Khurasan, and Transoxiana, and that of the western regions of Morocco and Andalusia. In the eastern regions, the techniques used to create this distinctive form of geometric pattern reached maturity during the fifteenth century under the artistic patronage of the rival Timurid, Qaraqoyunlu, and Aqqoyunlu dynasties [[7]]. In the far western regions of the Islamic world, maturity came a full century earlier under the auspices of the Nasrid and Marinid dynasties. Although it is not known for certain, the methods used in creating multiple-level geometric patterns appear to have developed independently in both regions. What is certain is that these developments represent the last great outpouring of inspired creativity in the long and illustrious history of Islamic geometric pattern making. What is more, for those with an interest in self-similar geometry, these designs are of special relevance. Evidently, many hundreds of years before the relatively recent discovery of this area of geometry, Muslim designers developed and refined the geometric arts to a level that conforms to the modern concept of self-similarity: wherein "an object is said to be self-similar if it looks "roughly" the same on any scale" [[8]]. Indeed, the fourteenth and fifteenth century Islamic traditions of multiple-level geometric design may well be the earliest human examples of sophisticated self-similar geometry.

Three Types of Self-Similar Islamic Geometric Patterns

The self-similar geometric patterns of the eastern regions comprise two distinct types; the first of which is characterized by a primary repetitive geometric pattern, with a reduced scale secondary geometric pattern

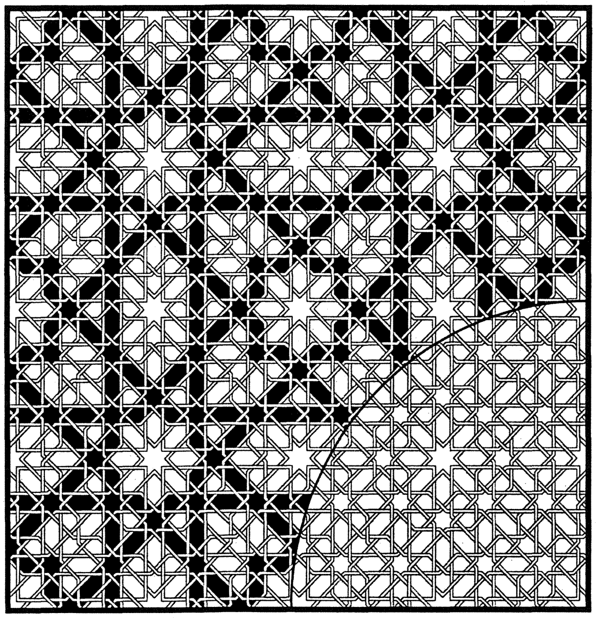

Figure 6: *Self-Similar Type C design from the Patio de las Doncellas in the Alcazar, Seville. Drawn both with, and without the color emphasis for the primary pattern.*

that exhibits the same geometric characteristics as the primary pattern, and *fills the entire background of the primary pattern*. I refer to these patterns as **Self-Similar Type A [Figure 4]**. The second type is made up of a primary geometric pattern, the lines of which have been widened to a proportion that allows for a secondary geometric pattern - with the same geometric characteristic as the primary pattern but at a reduced scale - to be *placed within the widened-lines*. These I refer to as **Self-Similar Type B [Figure 5]**. While the techniques used in creating the self-similar patterns of the western regions are essentially the same as those employed in the east, their aesthetic character is distinct and unique to Morocco and Andalusia. The self-similar patterns of this region *rely on color contrast to emphasize the primary design*. Unlike the designs of the eastern regions that work regardless of colorization, were the patterns of the western regions devoid of color they would appear as basic, if rather busy, geometric patterns without a multiple-level aspect. These designs from Morocco and Andalusia I refer to as **Self-Similar Type C [Figure 6]**. Each of these three design traditions are self-similar in that the geometric character of the primary pattern is always re-expressed at a smaller scale in the secondary pattern. This technique of fitting scaled down designs into the pattern matrices of larger scaled designs of the same geometric character can, in turn, be further applied to a third, fourth, and fifth level, ad infinitum. The fact that Muslim artists and designers limited themselves to only a single iteration of self-similarity is, I believe, due more to the constraints of the materials they were working with than any lack of creative imagination or geometric ingenuity.

We are fortunate that a wide variety of self-similar geometric patterns have survived to the present day. In the eastern regions, these examples are found in two media: architectural ornament, and as illustrations on paper in the Topkapi scroll, or *tumar* – an artist's reference collection of ornamental designs [9]. This important document includes a great quantity of designs for *Shatranji Kufi* calligraphy [10], *muqarnas* [11] and *rasmi* vaulting [12], standard two-dimensional geometric patterns, and several examples of Self-Similar Type A and B geometric patterns.

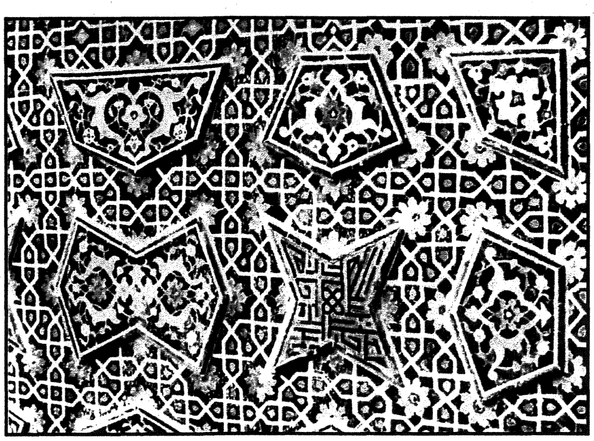

Figure 7: *Self-Similar Type B design from the Masjid-Jami, Isfahan.* Photo: courtesy of Antony Hutt and Leonard Harrow.

In the architectural record, most self-similar geometric designs were undertaken in cut-tile mosaic, called *muarraq* in Persian. This ceramic technique - first pioneered by the Seljuqs, further developed by the Muzaffarids, and taken to its highest expression during the Timurid and Safavid periods - has the benefit of being extremely precise. As such, it was ideally suited to the demanding nature of self-similar ornament. The combination of superlative cut-tile mosaics, vivid

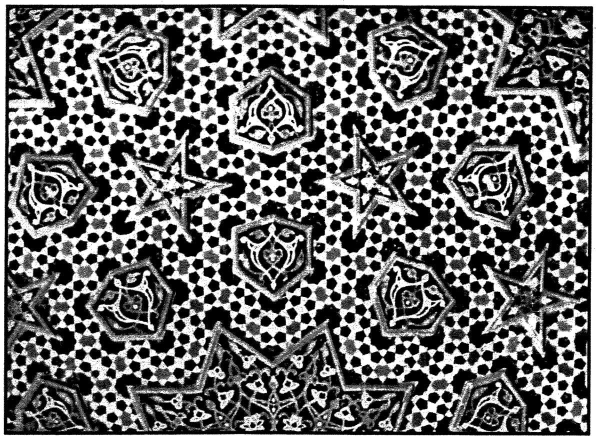

Figure 8: *5-fold Self-Similar Type B design from the Darb-i Imam, Isfahan.*

ceramic color, and ingenious self-similar geometric patterns created an ornamental tradition of overwhelming beauty. Of the many buildings with examples of self-similar cut-tile mosaic art, three are particularly noteworthy for both the quantity and quality of their designs. The Masjid-i Jami in Varzana (1442-44) is a magnificent Timurid mosque constructed during the reign of Shah Rukh, the son of Timur. The Darb-i Imam in Isfahan (1453-54) was built during the reign of the Qaraqoyunlu ruler Jahan Shah, and the 1475-76 additions to the Masjid-i Jami in Isfahan were built by the order of the Aqqoyonlu ruler Uzan Hasan. It is important to note that the work of the Qaraqoyunlu and Aqqoyonlu dynasties was carried out in the distinctive and dominant Timurid style. The self-similar designs found in these three buildings are all the more remarkable in that they are believed to be the creation of one specific artist: Sayyid Mahmud-i Naqash - one of the relatively few architectural ornamentalists in the long history of Islamic art who signed his name to his works [[13]]. The fact that Naqash's works are among the earliest examples of true self-similar geometric design found in the eastern Islamic world is an indication that he was likely one of the founders of this art form. Certainly Naqash deserves recognition not just as a great artist and designer, but also as a pioneer of self-similar geometry some 500 years ahead of his time. The work of Sayyid Mahmud-i Naqash in the Darb-i Imam includes the exquisite 5-fold Type A pattern pictured in **Figure 4**. His work in the Masjid-i Jami in Isfahan includes the elegant 4-fold Self-Similar Type B design shown in **Figure 7**; and his work in the Masjid-i Jami in Varzana includes the 3-fold Self-Similar Type B design pictured in **Figure 5**. It is remarkable that the earliest examples of this newly developed tradition of geometric art contained such diversity of style and symmetry. **Figure 8** is another example of the self-similar work of Sayyid Mahmud-i Naqash. This design from the Darb-i Imam is also a Type B design, but with 5-fold symmetry, and is exceptional for it ingenuity and beauty.

The Topkapi Scroll depicts seven self-similar geometric designs: five that are Type A, and two that are Type B [[14]]. It is not known who produced this design scroll, but according to Professor Gulru Necipoglu, the recognized authority on this subject, it was "probably compiled in the late 15th or 16th century somewhere in western or central Iran, possibly in Tabriz, which served as a major cultural capital under the Ilkhanids, the Qaraqoyunlu, and the Aqqoyunlu Turkmen dynasties, as well as the early Safavids. Its geometric designs in all likelihood were produced under Turkmen patronage, but an early Safavid date is also a possibility as the international Timurid heritage would still have been very much alive [[15]]." Pattern 28 in the Topkapi Scroll is a 5-fold Self-Similar Type A design that also depicts the underlying polygonal sub-grid used in the creation of the secondary design [**Figure 9**]. Similarly, an especially beautiful example of a Self-Similar Type B pattern is depicted in pattern 49 [**Figure 10**]. This self-similar design also has 5-fold symmetry, and is comparable in quality to the finest works of Sayyid Mahmud-i Naqash.

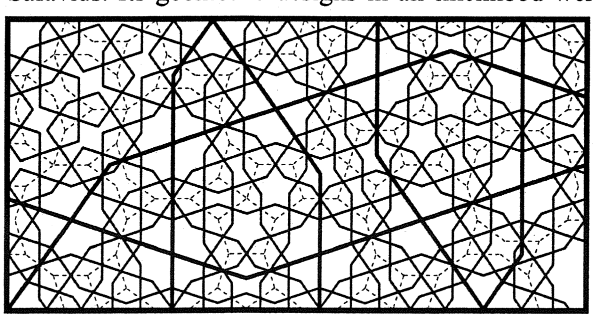

Figure 9: *5-fold Self-Similar Type A design from the Topkapi Scroll: No. 28. Redrawn for clarity.*

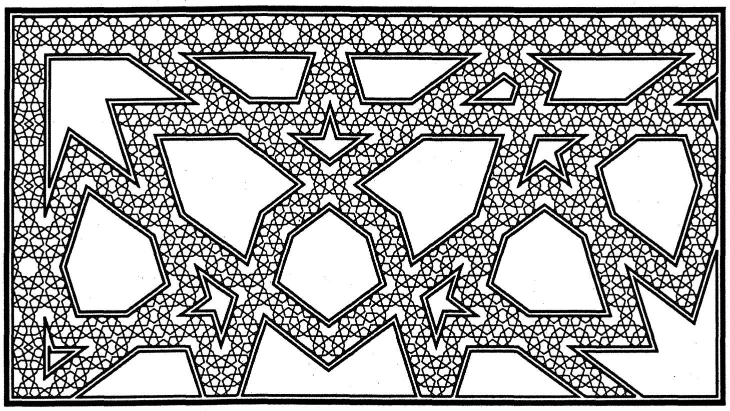

Figure 10: *5-fold Self-Similar Type B. design from the Topkapi Scroll: No. 49. Redrawn for clarity.*

Less is known about the individuals responsible for the fourteenth century development of Self-Similar Type C patterns in Morocco and Andalusia. There are no known design scrolls from this part of the Islamic world, and the Marinid and Nasrid artists responsible for their creation did not sign the ornamental panels that make use of self-similar geometry. As with the many examples from the eastern regions, the self-similar ornament from the west was universally undertaken in the medium of cut-tile mosaic – called *zillij* in the western Arabic dialect. Examples of Self-Similar Type C geometric patterns are found throughout Morocco and Andalusia. **Figure 6** shows a simple 4-fold example found at the Patio de las Doncellas in the Alcazar in Seville (c.1350). The two portions of this illustration demonstrate a key aspect of the western form of self-similar design: the need for color to differentiate their dual nature. Without the colored background, the primary design is lost within the matrix of the secondary pattern. The self-similar geometric designs of Morocco and Andalusia were almost always characterized by 4-fold symmetry. Both the Alcazar in Seville and the Alhambra in Granada (1333-91) have a variety of Type C zillij panels, and, indeed, many fine examples can be found throughout Morocco and Andalusia. This tradition also made occasional use of 5-fold symmetry, and such examples are found in the Madrasa of Bou Inaniya in Fez (1350-57), and the Attarine Madrasa in Fez (1323-25).

Polygonal Systems Used in the Creation of Self-Similar Islamic Geometric Patterns

Before detailing the historic techniques used to create all three types of self-similar geometric design, it is first necessary to very briefly describe the earlier methods used by pattern designers in the production of standard two-dimensional Islamic geometric patterns. It would be foolhardy to suggest that one, and only one, method was used historically in the creation of Islamic geometric patterns. However, one method stands out as the most ubiquitous in the creation of traditional geometric patterns: the polygonal technique. This method makes use of an underlying polygonal matrix, or sub-grid, from which the pattern lines can be derived. Once the creation of the geometric pattern has been accomplished, the initial polygonal sub-grid is discarded. This technique has tremendous merit over other techniques of pattern generation that have been proposed in recent decades [16]. From an art historical perspective, this is the only method for which there is documented proof that traditional designers used the system widely throughout the Islamic world [17]. The polygonal technique is the only method that allows for the creation of both simple geometric patterns and the most complex compound patterns, often made up

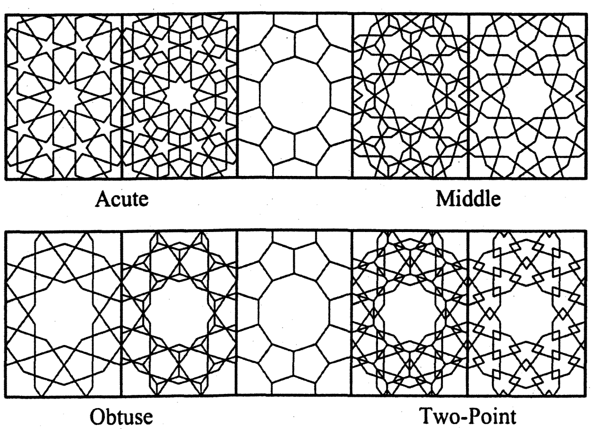

Figure 11: *Most common 5-fold polygonal sub-grid, and the four pattern families this technique produces.*

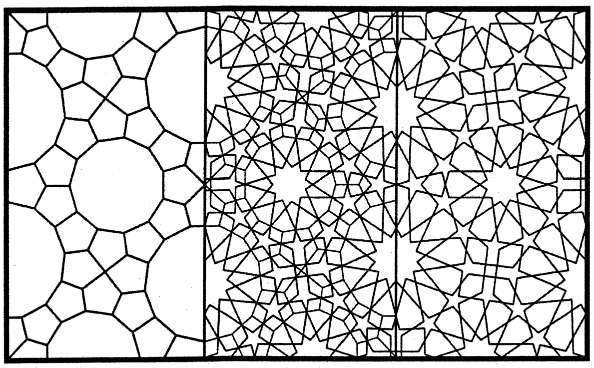

Figure 12: *Complex compound polygonal sub-grid with regular 11 and 13-sided polygons at the vertices of the repeat element, and the 'acute' pattern it produces.*

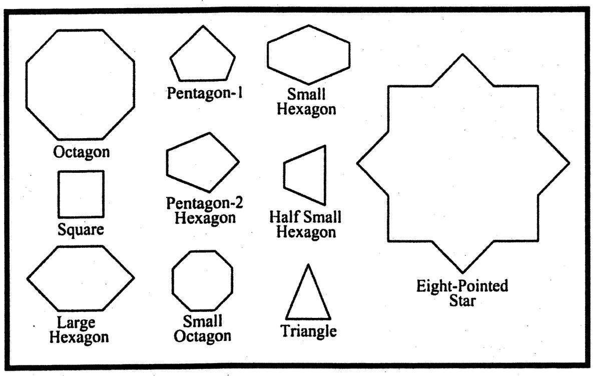

Figure 13: *Elements that comprise the 4-fold system of geometric pattern generation.*

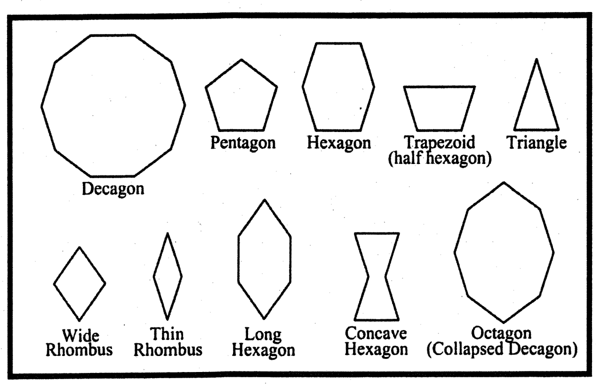

Figure 14: *Elements that comprise the 5-fold system of geometric pattern generation.*

of combinations of seemingly irreconcilable symmetries – such as patterns with regular 11 and 13-pointed stars at the vertices of the repeat units. No other proposed technique has attempted to explain the construction of these complex patterns, just as no other technique offers so much flexibility in creating the wide range of patterns found in this tradition. The polygonal technique has the further characteristic of allowing for the creation of all four principle families of Islamic geometric pattern regularly found throughout the Islamic world. These families are determined by variations in the angles of the crossing pattern lines that are applied to the initial polygonal sub-grid. In this way, four different families of pattern are derived from the polygonal sub-grid. These I have named *acute*, *middle*, *obtuse*, and *two-point*. **Figure 11** illustrates a well-used 5-fold polygonal sub-grid that can be traced back as far as the year 1000. This is the most widely used 5-fold sub-grid, and each of the four patterns this polygonal sub-grid creates was well known historically. This illustration demonstrates the diversity of patterns that a single polygonal sub-grid can produce. The lines of the *acute*, *middle* and *obtuse* patterns all cross the edges of the underlying polygons at their mid-point, and the different character of each of these three designs is due to the change in the angle of these crossing pattern lines. In the *acute* pattern, the angle of the crossing lines is 36°; in the *obtuse* pattern the angle is 108°; and in the *middle* pattern the angle is 72°. The *two-point* pattern derives its name from the fact that the patterns lines make use of two points on each polygonal edge rather than a single mid-point. Those Islamic geometric patterns that exhibit a more complex structure were almost always created from polygonal sub-grids that typically have a network of irregular polygons filling the space between the higher order regular polygons that occur at the vertices, centers, and occasionally along the edges of the repeat unit. **Figure 12** illustrates one of these complex and compound polygonal sub-grids, along with the very beautiful and ingenious *acute* pattern that it produces. However, it is important to note that the irregular and eccentric quality of such sub-grids does not lend itself to the creation of self-similar designs. For this, a series of far less complex and systematic polygonal sub-grids were employed.

Muslim designers invented several polygonal sub-grid systems that were used to construct geometric patterns. The hallmark of these systems is that each employs a set of polygonal elements in the creation of the underlying sub-grids. The combined use of different elements from a given set allows for an unlimited number of individual tessellations; each of which will create patterns in each of the four families. Of the three primary systems developed, one utilizes 4-fold symmetry, one 5-fold symmetry, and the third system derives patterns from tessellations made up of regular polygons: i.e. the triangle, square, hexagon, octagon, and dodecagon. The beauty of this systematic approach to pattern generation is that it is highly repetitive, easy to learn, and easy to put into practice. As this pertains to self-similar pattern construction, each of these systems has the further benefit of simultaneously working at more than a single scale of proportional reduction. **Figure 13** illustrates the ten polygons that make up the 4-fold system, and **Figure 14** illustrates the ten shapes that are common to the 5-fold system [18]. (The set of elements that make up the system of regular polygons are not illustrated, as they are self-evident.) Generally, the more members of the set used to create a polygonal sub-grid, the more complex the resulting pattern.

The Construction of Self-Similar Type A Patterns

The Darb-i Imam cut-tile mosaic panel shown in **Figure 4** is an ideal design for examining the techniques used in the construction of 5-fold Self-Similar Type A designs. **Figure 15** illustrates the primary pattern along with its underlying polygonal sub-grid. **Figure 16** demonstrates how scaled-down polygonal sub-grid elements are added to each edge length of the primary polygonal elements: the ratio (1:8.480) determined by fitting two decagons, separated by two pentagons, on the short edge length. It is interesting to note that the three edge lengths that comprise the polygons of the primary pattern have a phi proportional relationship. The next step in this design process is to simply fill in the rest of each primary pattern element with further reduced elements from the 5-fold system. **Figure 17** illustrates the application of the secondary polygonal sub-grid throughout the pattern matrix of the primary design, and **Figure 18** shows the application of the secondary pattern into the secondary polygonal sub-grid matrix.

That this very particular technique was used historically is confirmed in the Topkapi scroll. Pattern 28 from this scroll [**Figure 9**] makes use of small red dots to distinguish the underlying polygonal sub-grid of the secondary pattern. A series of otherwise invisible "dead drawing" lines were scribed with a steel point into the surface of this design [19]. These served as a means for the initial laying out of the underlying polygonal matrix necessary for the creation of the primary pattern. Between the scribed "dead drawing" lines and the fine red dotted lines, both the primary and secondary patterns are clearly shown to originate from the same self-similar geometric structure. This one example from the Topkapi scroll demonstrates the importance of this document in revealing the traditional techniques employed in the creation of Islamic self-similar geometric patterns.

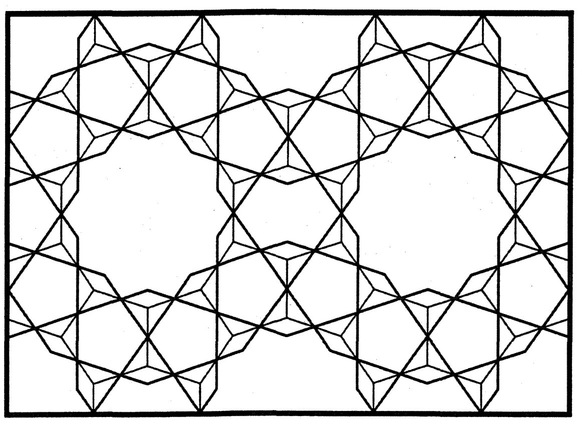

Figure 15: *The 5-fold polygonal sub-grid and 'obtuse' primary pattern used in the Self-Similar Type A design from the Darb-I Imam, Isfahan.*

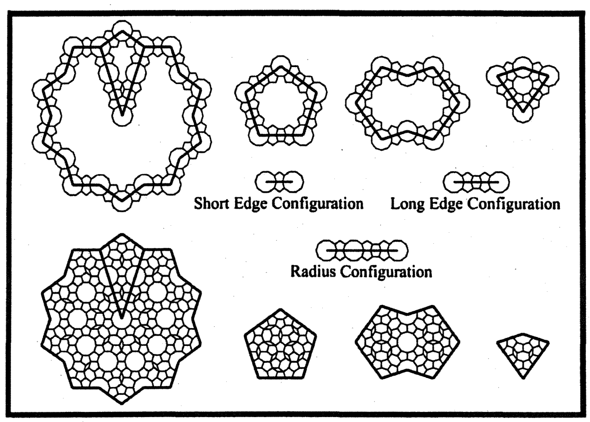

Figure 16: *Primary 5-fold pattern elements with added secondary polygonal sub-grid elements. Note: the edge and radius lengths have a phi relationship.*

The Darb-I Imam 5-fold example illustrated in **figures 15-18** places a secondary *obtuse* pattern onto an *obtuse* primary pattern. The regularity at both scales of the 108° angles that characterize the *obtuse* family results in true self-similarity in the completed design. Yet the 5-fold example from the Topkapi Scroll shown in figure 8 places a *middle* secondary pattern into an *obtuse* primary pattern: thus combining the respective characteristics of these two pattern families. Is this example still self-similar? The fact that the polygonal elements that make up both the primary and secondary sub-grids are identical in every respect save their size means that this pattern is certainly self-similar in its formative construction and underlying logic, if not strictly in its finished representation.

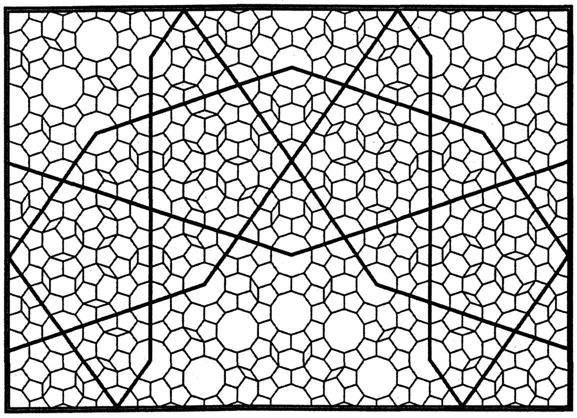

Figure 17: *The application of the secondary polygonal sub-grid elements to the primary pattern for the Self-Similar Type A design from the Darb-I Imam, Isfahan.*

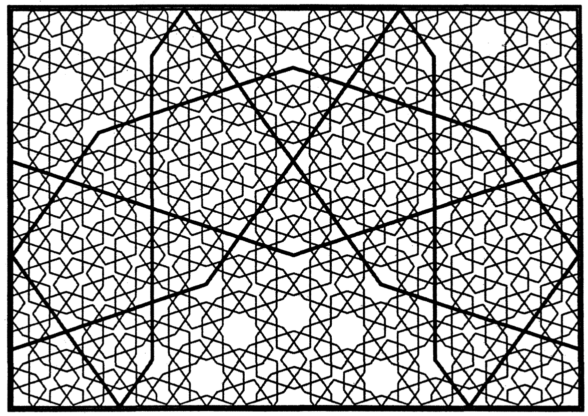

Figure 18: *Resulting 5-fold Self-Similar Type A design, with two iterations of 'obtuse' pattern from the Darb-I Imam, Isfahan.*

The Construction of Self-Similar Type B Patterns

As stated above, the Type B design illustrated in **Figure 10** was originally found in the Topkapi scroll. **Figure 19** illustrates the primary pattern along with its underlying polygonal sub-grid. **Figure 20** shows how the primary pattern lines can be widened so that they comprise a limited number of individual elements that share common features with the 5-fold polygonal system: for example, the proportions of the triangle and rhombus. **Figure 21** isolates each of these common elements and adds the smaller scaled secondary polygonal sub-grid elements from the same 5-fold system. In this case, the designer determined the scale of the reduced secondary polygonal elements by placing edge-to-edge decagons at each vertex of the rhombus (ratio=1:13.036). The completed secondary polygonal sub-grid is produced by then

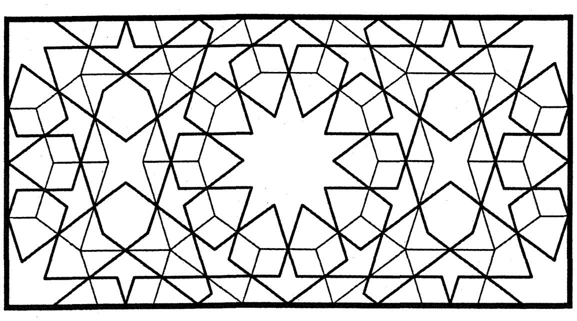

Figure 19: *5-fold 'acute' primary pattern, with its underlying polygonal sub-grid, from the Topkapi scroll: no. 49.*

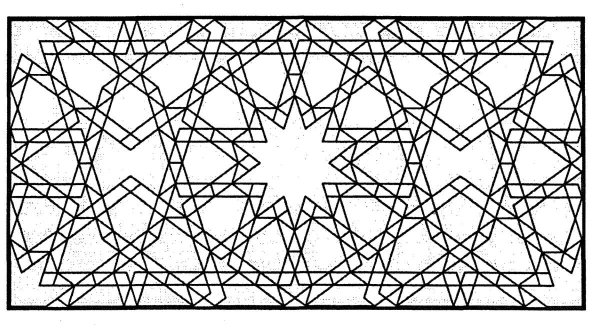

Figure 20: *Lines of the primary pattern widened to a thickness that allows for the further application of the secondary polygonal sub-grid elements.*

placing the same sized decagons at the vertices of the other four widened-line elements, and filling in the remaining space with additional shapes from the 5-fold system. Once again we see how the phi proportions of the edge lengths of the primary widened-line design accommodate the proportions of the scaled down polygonal elements for the secondary pattern. **Figure 22** applies the secondary polygonal sub-grid throughout the primary pattern. From this the entire self-similar pattern can be constructed. This example from the Topkapi Scroll uses an *obtuse* secondary pattern within the widened-lines of an *acute* primary pattern. The secondary pattern could just as easily also be made from the *acute* family.

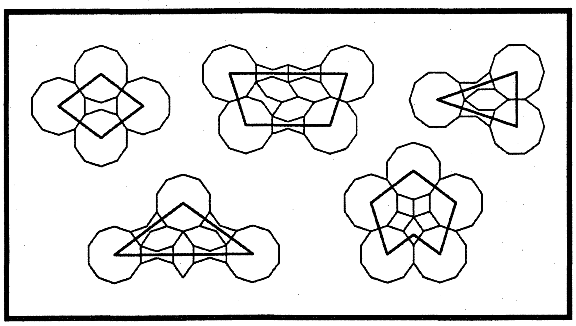

Figure 21: *The application of the secondary polygonal sub-grid elements to the individual widened-line segments that make up the primary pattern.*

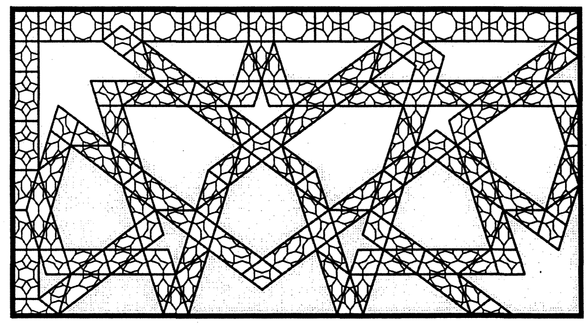

Figure 22: *The application of the individual widened-line segments, with the associated polygonal sub-grid elements, to the primary pattern.*

The Construction of Self-Similar Type C Patterns

Figure 6 illustrates a cut-tile mosaic panel from the Patio de las Doncellas in the Alcazar in Seville. **Figure 23** represents the primary pattern along with the polygonal sub-grid elements that combine to create this design. These sub-grid elements are members of the 4-fold system of geometric pattern generation. **Figure 24** demonstrates how the secondary polygonal sub-grid elements have been applied to the primary pattern lines [20]. Secondary octagons have been placed at each of the vertices of the primary pattern: their reduced size determined by the edge-to-edge placement around each eight-pointed star of the primary design (ratio=1:4.828). In similar fashion to the technique illustrated in Figure 16, further polygonal sub-grid elements are added to these initial secondary octagons, making up the completed polygonal sub-grid – or tessellation. Placing eight-pointed stars inside these octagons is the first step in creating the secondary pattern shown in **Figure 25.** The placement of the higher order polygons (in this case octagons) at the vertices of the primary pattern is significant in that Self-Similar Type C designs require stars at each vertex of the primary pattern. Furthermore, these stars must have lines that are parallel to those of the primary pattern in order for the colored background technique to work. In emphasizing the primary pattern through applied color, the secondary pattern lines are widened, and made to interweave with themselves.

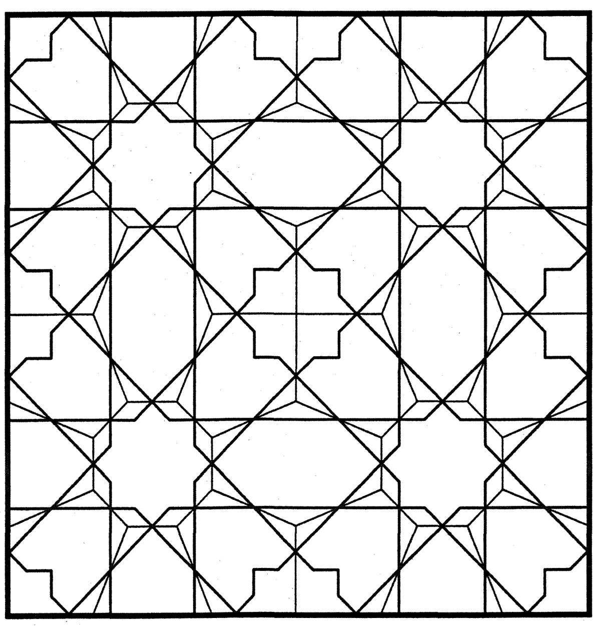

Figure 23: *Primary pattern with its polygonal sub-grid from the Patio de las Doncellas in the Alcazar, Seville.*

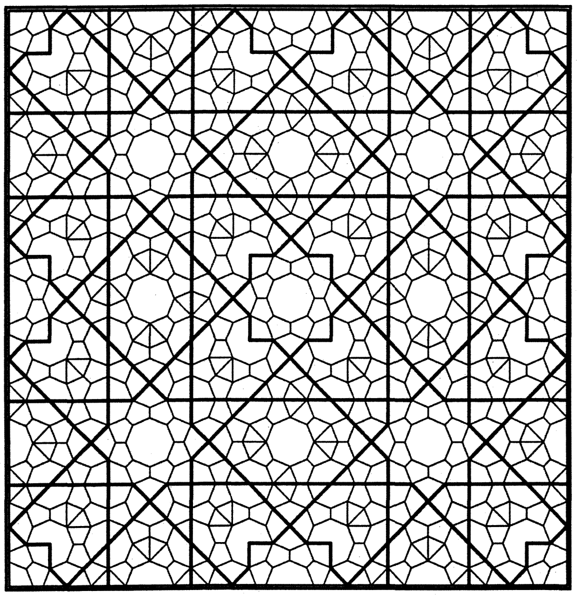

Figure 24: *Application of the secondary sub-grid elements to the primary pattern from the Patio de las Doncellas in the Alcazar, Seville.*

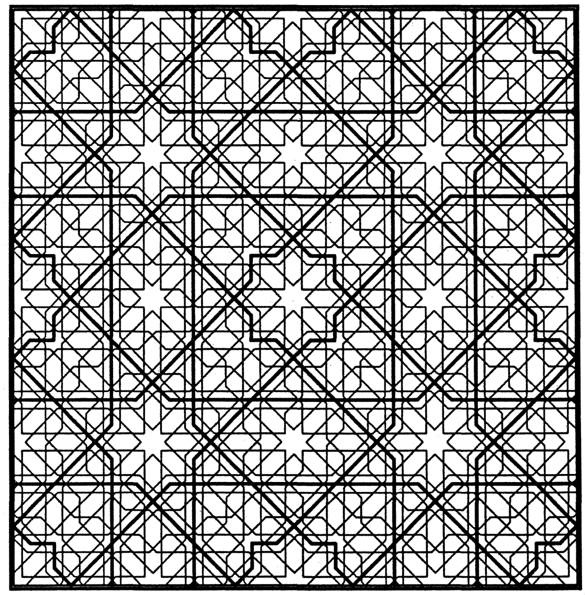

Figure 25: *Primary and secondary patterns before widening and interweaving of the secondary lines, and subsequent application of color.*

Potential Realms for Further Exploration

Despite the tremendous diversity of self-similar patterns left to us by Muslim designers, there is certainly a wealth of potential for new work in this field. The techniques outlined above are simple, yet their results can be very beautiful and inspiring. Moreover, the inexhaustible potential for further design innovations is pregnant with possibilities. Within two-dimensional space, there are further polygonal sub-grid systems

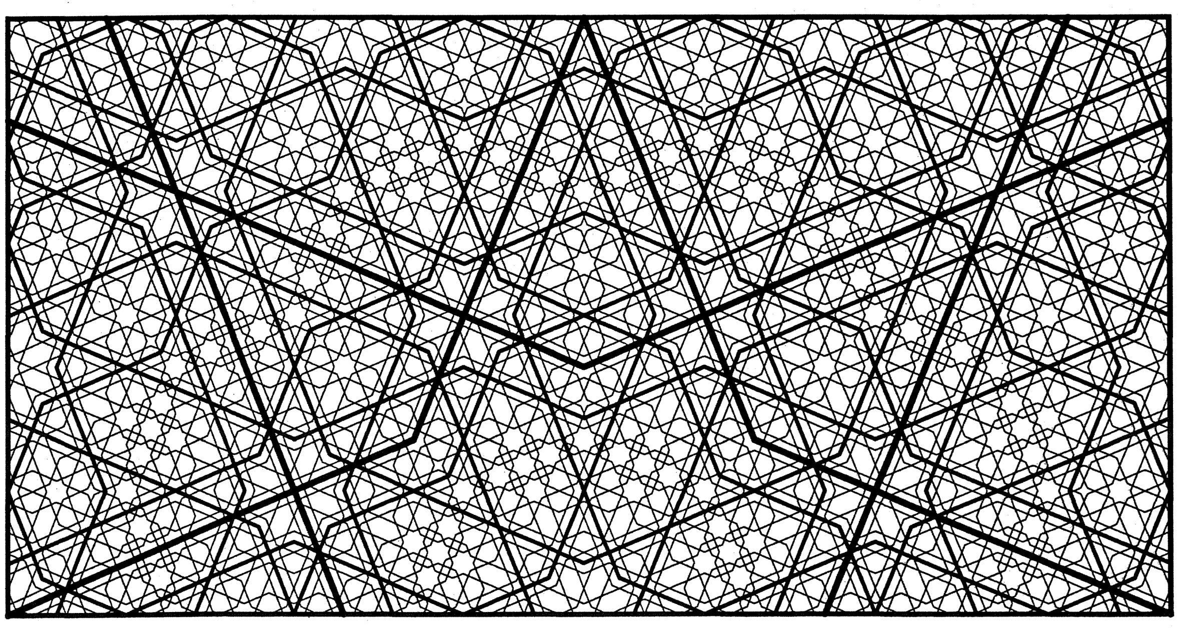

Figure 26: *Three-level Self-Similar Type A design made from the 4-fold polygonal sub-grid system of pattern generation.* Designed by the author.

that can be applied to self-similar pattern generation: there is the possibility of exploring a greater number iterations of self-similarity [**Figure 26**] – even infinite iterations through animation [[21]]; and there is the interesting variation of applying these techniques to aperiodic space filling [**Figure 27**]. What is more, the recent interest in applying Islamic geometric patterns to non-Euclidean space can be extended into the application of such self-similar patterns into both hyperbolic and spherical surface coverage [[22]]. Clearly this is an area that can challenge the imagination and stir the heart with its innate and pure form of beauty.

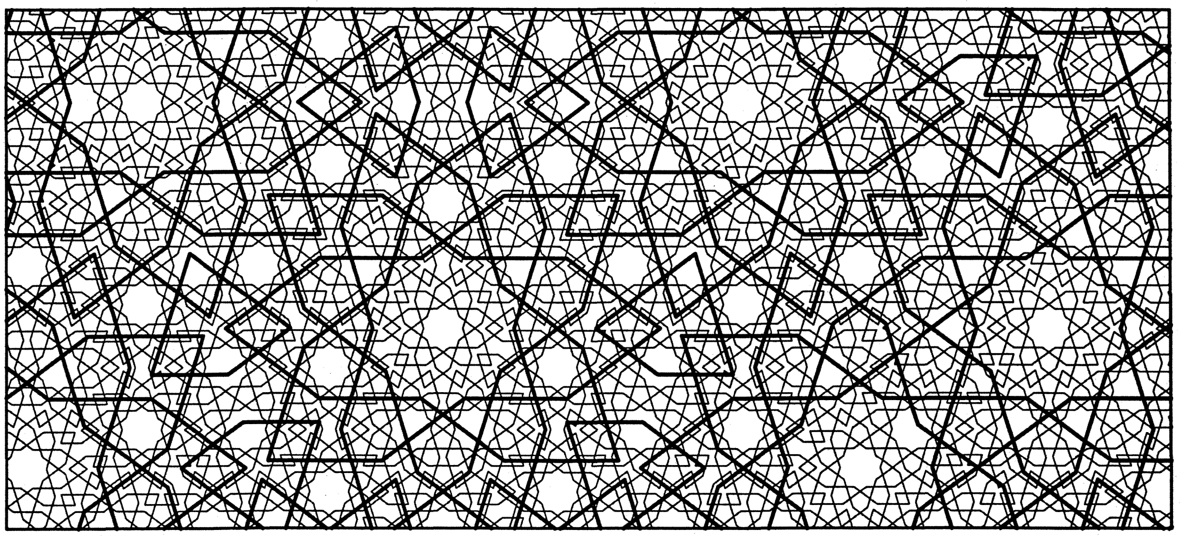

Figure 27: *Aperiodic Self-Similar Type A design made from the 5-fold system of pattern generation. Both the primary and secondary levels are of the 'middle' family of Islamic geometric patterns.* Designed by the author.

References
(For complete references text, please visit the author's web site: see above)

[[1]] Unlike earlier Umayyad window grills, the example from the Ibn Tulun has a multiple-level quality.
[[2]] The Persian term *yezdi bendi* translates as the "work of Yezd".
[[3]] *Kufi* is among the earliest Arabic scripts. Its many forms are characterized by an angular quality.
[[4]] The *alif* is the first letter of the Arabic alphabet. It is an ascender that is made from a single vertical stroke.
[[5]] *Thuluth* is one of the principle cursive Arabic scripts, and widely used in both the book arts and epigraphy.
[[6]] Jay Bonner, *Islamic Geometric Patterns*, White Cloud Press (forthcoming).
[[7]] Gulru Necipoglu, *The Topkapi Scroll*; pp. 35-39.
[[8]] Eric Weisstein's World of Mathematics: http://mathworld.wolfram.com/search/
[[9]] Gulru Necipoglu, Ibid. The historic significance of the Topkapi Scroll has been fully detailed in this work.
[[10]] *Shatranji Kufi* is an extreme angular script wherein each letter is determined by the orthographic grid.
[[11]] A system of Islamic vaulting made up of many small clustered arched elements.
[[12]] Also referred to as star-net vaulting: made from an array of overlapping arches, creating a star at the apex.
[[13]] Antony Hutt and Leonard Harrow, *Iran 2*, Scorpion Publications Ltd., 1979, pp. 61-65.
[[14]] Gulru Necipoglu, Ibid. Scroll references in this paper use the numbering system of Professor Necipoglu.
[[15]] Gulru Necipoglu, Ibid. pp. 37-38.
[[16]] Jay Bonner, Ibid. This work details a variety of techniques used to create Islamic geometric patterns.
[[17]] Jay Bonner, Ibid. Proof for the traditional use of this system is detail in the author's forthcoming book
[[18]] Jay Bonner, Ibid. The construction of these elements is detailed in the author's forthcoming book.
[[19]] Gulru Necipoglu, Ibid., p. 239; p. 249, no. 28; p. 300, no. 28.
[[20]] Jean-Marc Castera, *Arabesque: Decorative Arts in Morocco*; Arc Edition, 1999, pp. 276-277.
[[21]] Jean-Marc Castera, Ibid., pp.276-277.
[[22]] Jay Bonner, *Geodazzlers*, Design Science Toys. http://www.dstoys.com/gd.html
Craig Kaplan, *Computer Graphics and Geometric Ornamental Design*. Ph.D. University of Washington, Seattle, 2002, pp. 67-90. http://www.cgl.uwaterloo.ca/~csk/phd/