

On the Shapes of Water Fountains and Times Tables

Stephen Eberhart
Department of Mathematics
California State University, Northridge
Northridge, CA 91330-8313

When I began teaching mathematics at Waldorf schools (newly recruited from being an orchestra musician), I relied heavily on the books of one of their past master teachers, Hermann von Baravalle, in one of which he suggests having students draw a sequence of parabolas from a common point by connecting dots up 3 units, up 1 unit, then down 1, 3, 5, 7, etc., and going left or right 1 unit each time for one pair of curves, 2 units for a second pair, and so on, calling the result a fountain. I tried this, and was immediately nailed by the typical back-row pupil to whom I have since come to be so grateful: "Fountains aren't flat on top!" A private session with the physics teacher and some resulting expressions for conservation of kinetic and potential energy later, we found that if individual water jets (emanating from a spherical nozzle under equal pressure, viewed in profile) are parabolic in Galileian ideal, then the locus of their vertices is a 2 x 1 sideways ellipse (passing through nozzle and peak) and their envelope of all jets is the parabola tangent to that ellipse at peak – nice!

Meanwhile, apparently unrelated, I observed a 4th grade teacher repeating some wisdom he'd borrowed from another past master, William Harrer, by drawing the usual 45 triangular times table shape bent around into a sort of free-hand heart form. This seemed pleasant enough but highly arbitrary, so I put the question to various friends what the shape of a times table might more appropriately be (if not its usually Philistine 45 form). Right away, the water fountain came back as one possible suggestion (the curves now at least physically motivated); various star polygons also came into the mix when finite modular multiplication systems were included, and finally star polygons on Moebius strips (topologically, drawn in the projective plane with a central disk removed).