BRIDGES
Mathematical Connections
in Art, Music, and Science

Graphical Fractals Based on Circles

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Fractals such as the Mandelbrot set are constructed computationally, based on an equation. It is also possible to construct fractals graphically, based on a set of rules for progressing from one iteration to the next. The use of a computer drawing program such as FreeHand greatly aids this repetitive construction process. A variety of such fractals are constructed using the circle as the fundamental building block. In some of these, overlapping circles are employed, while in others, the circles do not overlap, but touch tangentially. An example of the latter is shown in Figure 1.

The creative element enters in choosing matching rules that yield an esthetically-pleasing fractal. As with any geometric design, the appearance of a fractal generated in this manner can be varied greatly. There are virtually limitless choices in coloring, rendering, and choice of background. My digital print "Circle Composition II", exhibited at this conference, depicts a fractal in which the circles were rendered using PhotoShop to suggest perhaps an alien technological construction.

A particular mathematical tool used in several of these designs is the Descartes circle theorem, which relates the radii of four mutually tangent circles. In addition, carrying out some algebra on infinite series allows determination of scaling factors that just allow touching of circles in the limit of an infinite number of iterations. In the case of the fractal shown in Figure 1, this process yields a scaling factor between successive iterations of exactly the square of the Golden ratio.

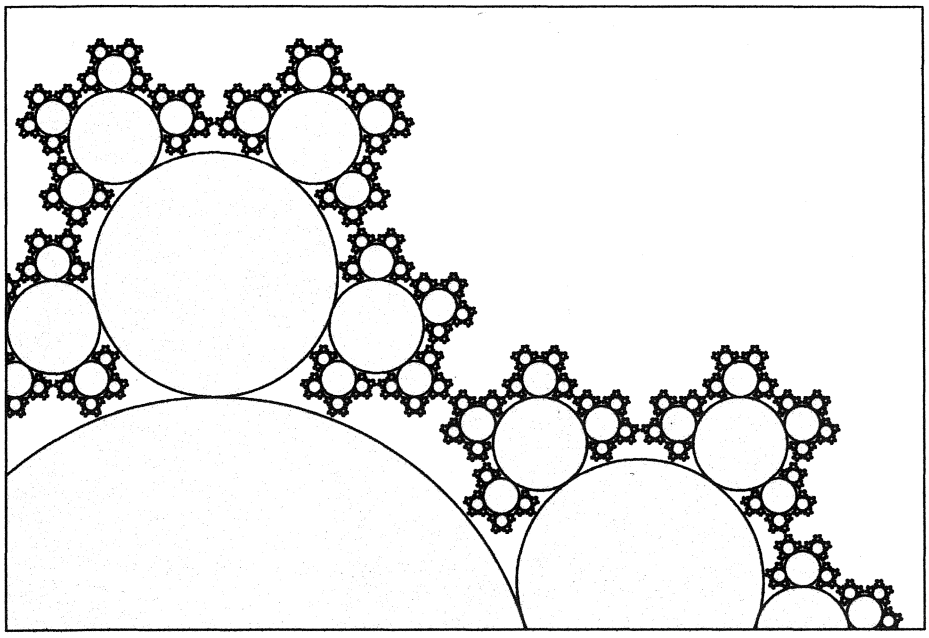

Figure 1. *A portion of a graphical fractal constructed from circles that touch tangentially. The overall fractal has five-fold rotational symmetry, and the scaling factor from any given circle to the next smaller circle is the square of the Golden ratio.*