BRIDGES
Mathematical Connections
in Art, Music, and Science

Beautiful Mind—Elegant Bridge

Daniel F. Daniel
Southwestern College
Winfield, KS 67156, U.S.A.
E-mail: ddaniel@sckans.edu

Abstract

John Forbes Nash, Jr., a mathematician, won the Nobel Prize for Economics in 1994. His work on *n*-person game theory challenged earlier work by von Neumann and Morgenstern as well as that by Adam Smith. Smith's understanding of supply and demand, with distribution of goods governed by the "invisible hand" of market forces, had governed economic theory since the eighteenth century. Nash has contributed significantly to the New Economy with work which creates another bridge from mathematics to economics. Nash was the subject of a recent biography and of a movie both titled *A Beautiful Mind.*

1. Games

As new economic theory is formulated in multiple postmodern forms, economists such as Schumpeter and Hyack are being praised for having anticipated what is often called the hermeneutic turn by humanists. The discovery that socialism and capitalism may co-exist within the same economy no longer shocks many.

The turn that Nash brought to Adam Smith's model of supply and demand, however, continues to intrigue the economics community. The solutions that von Neumann had once dismissed as "trivial," an ultimate mathematical insult, are steadily altering the ancient view of markets and of market-making. That games can be so pivotal will hardly come as a surprise to the groups of mathematicians who daily gather for coffee and bridge, chess, Go, or the Game of Life in afternoon sessions in departments around the country.

John Conway has suggested that the concept of surreal numbers, "the idea of a lifetime," came to him while playing Go one day. Nash had himself invented a game, which is still played at Princeton. Nash was 21 when he produced the 27-page dissertation, which has profoundly impacted economics. Even the best price theorists had acknowledged that despite Adam Smith's faith in the "invisible hand" of the market, real world markets often produced inefficiencies. Nash reformulated the market as a game. His central insight was the proof that every economic game has an equilibrium point. That is, a point beyond which no player would want to change his strategy.

I do confess to being more fascinated by the film's image of Nash working on the Riemann hypothesis on the library windows or in his watching the pigeons in the quad in an attempt to derive a natural algorithm, than in the brief attention given to game theory. (The film's illustration was Nash's game theory being used to pick up girls in a bar.) Even the dark, schizophrenic heart of the movie must have seemed sad to anyone interested in the intellectual development of

mathematical ideas. But film criticism aside, we did have a brief chance to celebrate a mathematician and mathematics. This is a continuing rarity in the culture.

Regardless of the moviemakers who fictionalized Nash's life, Nash has come to symbolize with or without Hollywood the mathematizing of economics as well as the application of game theory to market analyses of all sorts—from auctions to arms races, game theory has become a ubiquitous mathematical tool. According to biographer Sylvia Nasar:

> "The prize itself was a long-overdue acknowledgement by the Nobel committee that a sea change in economics, one that had been under way for more than a decade, had taken place. As a discipline, economics had long been dominated by Adam Smith's brilliant metaphor of The Invisible Hand. Smith's concept of perfect competition envisions so many buyers and sellers that no single buyer or seller has to worry about the reactions of others. It is a powerful idea, one that predicted how free-market economies would evolve and gave policymakers a guide for encouraging growth a guide for encouraging growth and dividing the economic pie fairly. But in the world of megamergers, big government, massive foreign direct investment, and wholesale privatization, where the game is played by a handful of players, each taking into account the others' actions, each pursuing his own best strategies, game theory has come to the fore." [1]

While certainly not replacing Adam Smith, Nash reoriented the thinking about maximizing the results based on human behavior. By focusing on the individual he showed the possibility for mutual gain. Just as Einstein did not replace Newtonian mechanics, he did give us a new generation of thought about the structure of the universe. In the introduction to his dissertation, Nash writes,

> Von Neumann and Morgenstern have developed a very fruitful theory of two-person zero sum games in their book Theory of Games and Economic Behavior. This book also contains a theory of two-person games of a type which we would call co-operative. This theory is based on an analysis of the interrelationships of the various coalitions which can be formed by the players of the game. Our theory, in contradistinction, is based on the absence of collations in that it is assumed that each participant acts independently, without collaboration or communication with any of the others. [2]

As Naser acknowledges, Nash's insight once explained "seems obvious." But in offering an image of a decentralized decision making that is coherent economically, an economics tool at once sophisticated and revolutionary has been provided economists.

Naturally mathematicians are not universally delighted by Nash's entrance into what economist Paul Samuelson called the "swamp of *n*-person game theory." Conway has called Nash's work on embedding any Riemannian manifold in a Euclidean space, "one of the most important pieces of mathematical analysis in this century." [3] Certainly Nash's reputation among mathematicians does not reside in his rearticulation of game theory. Norman Levitt, from Rutgers University, notes:

> Other work of Nash's is more important and striking. In particular, he is best known as the creator of the Isometric Embedding Theorem, which shows that arbitrary Riemannian manifolds can be realized, metrically, as submanifolds of Euclidean space. This is a fundamental result in Differential Geometry, a field far removed from game theory. The Nobel Hardly makes up for the fact that Nash was denied a Fields Medal of this work—in retrospect, a defective judgment. The excessive emphasis on the game-theory paper is one of the ironies of Nash's sudden celebrity. [4]

Even today there is no accounting for taste. Not yet at least

2. Economics

Of course this paper's title anticipates the coming Bridges Conference, offers a symbolic response to Carlo Sequin's quest for a bridge for Bridges, and considers the bridge between Nash's game theory and the reformation of contemporary economics.

Social Scientists can be as put off by game theory I have discovered as can mathematicians be disappointed in the attention an essentially minor work in mathematics has attracted. In interviewing a neo-liberal economist trained at the University of Chicago for an article on Chilean economics, I happened to ask him what he thought Nash's role had been in redefining economics. Richard Leftwich's response was quick and absolute: "I think economics took a major step backwards when it mathematized the economics discipline. Nash didn't care about economics. He cared about mathematics." [5]

Regardless of Leftwich's enmity to mathematizing in general and to game theory in particular, very few seem anxious to put the game back in the box. Nasar quotes Avanish Dixit, a game theorist at Princeton:

> "Concepts, terminology, and models from game theory have come to dominate many areas of economics. At last we are seeing the realization of the true potential of the revolution launched by von Neumann and Morgenstern. And because most economic applications of game theory use the Nash equilibrium concept, Nash is the point of departure." [6]

She adds, "The current generation of economic policymakers—including Lawrence Summers, [former] Undersecretary of the Treasury [current president of Harvard], Joseph Stiglitz, [former] chairman of the Council of Economic Advisors [current fellow of the Brookings Institute], and [former]Vice-President Al Gore—are steeped in the stuff which they say is useful for thinking about everything from budget proposals to Federal Reserve policy to pollution cleanups." [7]

In a recent central bank colloquy, Harvard President Lawrence Summers presented a paper which indicated his growing reliance on game theory. At Harvard, Summers is now dealing with the issue of intellectual property. To indicate some possible solutions to the creation of incentives for innovation and of equity with the university's resources, Summers cites the recent work of faculty member Michael Kremer. Kremer had done work on public purchase of patents at auction and of shifting some public research from effort-oriented to result-oriented processes, that is for example, holding contests for private companies to develop vaccines instead of funding research directly. This is especially intriguing in its attempts to develop institutions that have the advantages of market competition and of natural monopolies.

Summers acknowledged to the bankers and economists assembled that "It will be no surprise to you that at least one of the authors [DeLong of Berkeley is the other author]has been thinking very hard about the role the heavily-endowed non-profit educational institutions have to play in attempting to resolve the dilemmas of innovation and intellectual property in the 'new economy.'" [8]

Therefore in auctions, contests, and the creation of new approaches to incentive-creation for a more efficient system of basic research and discovery, we begin to mark an even greater alteration

of the marketplace than that imagined by Adam Smith. The language of the strategic game, as current texts now examine the relationships between corporations and consumers, is here to stay.

3. Conclusion

More than in most arenas of creativity, mathematicians hold it axiomatic that genius and youth are inextricably united. As a humanist I would be remiss not to offer Nash's own poignant observations about his schizophrenia and his mathematical productivity.

> ". . .at the present time I seem to be thinking rationally again in the style that is characteristic of scientists. However, this is not entirely a matter of joy as if someone returned from physical disability to good physical health. One aspect of this is that rationality of thought imposes a limit on a person's concept of his relation to the universe. . . .

Statistically, it would seem improbable that any mathematician or scientist, at the age of 66, would be able through continued research efforts, to add much to his or her previous achievements. However I am making the effort and it is conceivable that with the gap period of about 25 years of partially deluded thinking providing a sort of vacation my situation may be atypical. Thus I have hopes of being able to achieve something of value through my current studies or with any new ideas that come in the future." [9]

And who would question John Nash's optimism?

References

[1] Nasar, S. (1998), *A beautiful mind.* New York: Simon and Schuster.

[2] Kuhn, H.W. & Nasar, S. (Eds). (2002) *The essential John Nash.* Princeton, New Jersey: Princeton University Press.

[3] Quoted in Kuhn & Nasar,(2002). *The essential John Nash.*

[4] Printed as a response to Wright, R. Nash equilibrium" in *Slate.* Retrieved on May 23, 2002 from http://find.slate.msn.com/default.aspx?id=2063457

[5] Interview with R. Leftwich. April 2002.

[6] Nasar, S. (1998), *A beautiful mind.* Also in Nasar, S. (2002) A story about the mystery of the human mind, in 3 acts: genius, madness, reawakening (MSNBC). Retrieved on April 11, 2002 from http://www.msnbc.com/news/727643.asp

[7] Nasar, S.(1998). *A beautiful mind.*

[8] DeLong, J.B. & Summers, L.H. (Fourth Quarter 2001). "The new economy": background, historical perspective, questions, and speculations. *Federal Reserve Bank of Kansas City: Economic Review*, 4, 29-59.

[9] Kuhn, H.W. & Nasar, S. (2002). *The essential John Nash.*