Pure-Tone Approximation to Equal-Temperament:
How Close is Close?

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We, briefly, review the role of generated tuning systems in music theory and discuss several scale properties inherent in these generated systems. First explored in the ground breaking works by Carey and Clampitt (1989, 1996), the seemingly unrelated properties of: 1) Dual Symmetry [Clampitt (1997)], 2) Myhill's Property [Clough and Myerson (1982, 1983)], 3) Spectra Widths [Carey and Clampitt (1996), and Clampitt (1997)], and 4) the Three Gap Theorem [Swierczkowski (1958) and Sos (1958) among others] will be shown to give equivalent descriptions of, so-called, *well-formed musical scales* - which have been shown to provide a single structural basis in music theory. The scale properties of generated systems will be discussed in the context of the *well-formed scales* of Carey and Clampitt. We will, then, introduce and discuss equivalent measures, so-called *equal-tempered system variations* (ETS variations), based on the properties of *well-formed scales*. It will be shown that ETS variations determine the "closeness" of a musical scale to a given equal-tempered scale and may be used to generate pure-tone approximations to equal-temperament.