BRIDGES
Mathematical Connections
in Art, Music, and Science

A Museum Dedicated to the Concept of Harmony and the Golden Section

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Abstract

It is stated in the article a concept of the Museum of Harmony and the Golden Section as unique history-nature-science-art museum, collection of the Nature, Science and Art works based on the Golden Section.

1. The Concept of Harmony and the Golden Section

Throughout history people have desired to surround themselves with beautiful things. At some point the question arose: What is the basis of beauty? Ancient Greeks developed the science of *aesthetics* as a way of analyzing beauty, believing that harmony was its basis. Beauty and truth are interrelated: an artist searches for truth in beauty, and a scientist for beauty in truth.

Is it possible to compare the beauty of a sculpture, a temple, a picture, a symphony, poem, or nocturne? If a formula could be found, then the loveliness of a chamomile flower and that of a naked body could be measured and compared. The well-known Italian architect Leone Battista Alberti spoke about harmony as follows: "There is something greater, composed from combination and connection of three things (number, limitation and arrangement), something that lights up all face of beauty. And we called it by Harmony, which is, undoubtedly, the source of some charm and beauty. You see assigning and purpose of Harmony to arrange the parts; generally speaking, different under the nature, by certain perfect ratio so that they met one another creating a beauty ... It encompasses all human life, penetrates through the nature of things. Wherefore everything that is made by Nature, all this is measured by the law of harmony. Also no for the Nature of the greater care, than that everything created by it was perfect. It is impossible to achieve this without Harmony, wherefore without it the greatest consent of the parts is disintegrated." (the quotation is taken from [1], p. 191; English translation was made by A. Stakhov).

There are many well-known "formulas of beauty" such as certain geometrical shapes: square, circle, isosceles triangle, and pyramid. One unique mathematical proportion is called the Divine Proportion, Golden Section, Golden Number, or Golden Mean. The Golden Section and its related Fibonacci numbers permeate the history of art. Examples of well known works exhibiting this proportion are the Great Cheops Pyramid of Egypt, the Parthenon in Athens, Greek sculpture, the "Mona Lisa" by

Leonardo da Vinci, paintings by Rafael, Shishkin, and the modern Russian artist Konstantin Vasiljev, Chopin's etudes, music of Beethoven, Tchaikovsky, and Bartok, and "Modulor" by Corbusier.

2. Contents of the Museum of Harmony and Golden Section

The Museum of Harmony and Golden Section contains a vast collection of information on the Golden Section in nature, science, and art. In virtual form, the Museum can be seen on the Web at http://www.goldenmuseum.com/. The main goal of the museum is given in the introduction: "The 'Golden Proportion' is a mathematical concept and its analysis is first of all a problem of science. But it is a criterion of Harmony and Beauty, and this is already category of Art and Aesthetics. And our Museum, which is dedicated to analysis of this unique phenomenon, is doubtlessly, scientific museum dedicated to the analysis of harmony and beauty from the mathematical point of view."

The Museum contains two main parts: *cognitive* and *scientific*. The former part aims to acquaint all people—students, teachers, engineers, specialists in various areas of science, artists, musicians, and representatives of all arts—with surprising discoveries of ancient science: the Golden Section and its various applications. The scientific part of the Museum is aims to give information on modern scientific discoveries based on the Golden Section; in particular, about the "Mathematics of Harmony" and its applications to modern science.

The Museum consists of the following halls:

(1) Golden Section in a History of Culture
(2) Golden Section, Nature and Man
(3) Golden Section in Art
(4) Mathematics of Harmony
(5) Fibonacci Computers
(6) Fibonaccization of Modern Science
(7) Harmonic Education

3. Golden Section in the History of Culture

This hall of the Museum consists of the following exhibitions:

(1) What is the meaning of the Golden Section and Fibonacci Numbers?
(2) The Golden Section in the history of Ancient Art
(3) Fibonacci numbers and the Golden Section in the Middle Ages and Renaissance
(4) The problem of Harmony and Symmetry in the science of the 19^{th} century

Although the material of every exhibition is well known separately, the collection of facts concerning the Golden Section confirms the outstanding role it plays in the history of culture. Let's consider some new scientific evidence about the role of the Golden Section in the history of material culture.

3.1. Phenomenon of Ancient Egypt. Early in the 20^{th} century in Saqqara (Egypt), archaeologists opened the crypt in which the Egyptian architect Khesi-Ra (Khesira) was buried. Wooden panels covered with a magnificent thread were extracted from the crypt along with other materials. There were eleven of these boards; among them only five were preserved as the remaining panels were damaged because of moisture reaching the crypt.

Investigating Khesi-Ra panels the Russian architect Igor Shmelev made the following discovery: "But now, after the comprehensive and argued analysis by the method of proportions we get good causes to assert that Khesi-Ra's panels are the harmony rules encoded by geometry language.... So, in our hands we have the concrete material evidence, which shows us by "plain text" the highest level of abstract thought of the Ancient Egypt intellectuals. The artist cutting the panels with amazing accuracy, jeweler refinement and masterly ingenuity demonstrated the rule of the 'Golden Section' in its broadest range of variations. In outcome it was born the 'GOLDEN SYMPHONY' presented by the ensemble of the highly artistic works, which testifies not only ingenious talents of their creator, but also verifies convincingly that the author was to let in on the secret of harmony. This genius was of the 'Golden Business Craftsman' by the name of Khesi-Ra." (the quotation is taken from [2], p.49-50; English translation was made by A. Stakhov).

Another quotation from Shmelev's brochure, "Phenomenon of the Ancient Egypt" (1993), states: "It is necessary only to recognize that the Ancient Egypt civilization is the super-civilization studied by us extremely superficially and demanded on qualitatively new approach to development of its richest heritage.... The outcomes of researches of Khesi-Ra's panels demonstrate that the sources of modern science and culture are in boundless stratums of a history feeding creativity of the craftsmen of our days by great ideas, which a long time inspired aspirations of the outstanding representatives of mankind. And our purpose is doing not lose a unity of a binding thread." (the quotation is taken from [2], p. 55; English translation was made by A. Stakhov).

3.2. Mystery of the Egyptian Calendar. The Egyptian calendar, created in the 4^{th} millennium B.C., was one of the first solar calendars. The year consisted of 365 days, 12 months of 30 days each, and 5 holidays at the end of the year; thus it was the prototype of the modern calendar.

Several questions arise about the creation of this calendar. Why did the Egyptians choose 12 months? Why did each month have 30 days? Other calendars, such as the Mayan, consisted of 18 months with 20 days in each. Similar questions concern the Egyptian system of time and angle measurement, in which the number 60 seems to recur. Why is a circle circumference divided into 360 degrees ($2\pi = 360^\circ = 12 \times 30$)? Why did early astronomers consider that there were 12 "zodiacal" signs, though actually the Sun intersects 13 constellations? And, further, why did the Babylonian number system have as its base the number 60?

In analyzing these questions, we find that four numbers consistently arise: 12, 30, 60, and $360=12\times30$. In most ancient calendars originating in eastern and southeastern Asia, attention was given to the motions of the Sun, the Moon, and the two largest planets of the solar system, Jupiter and Saturn. Evidence suggests that the Jupiter calendar with its celestial symbolism of the 12-year animal cycle is connected to the rotation of Jupiter around the Sun, approximately 12 years (11.862 years). Saturn makes its full revolution around the Sun in approximately 30 years (29.458 years). Based on these numbers, the Ancient Chinese introduced the 60-year cycle of the solar system, during which Saturn makes two full revolutions around the Sun and Jupiter, five revolutions.

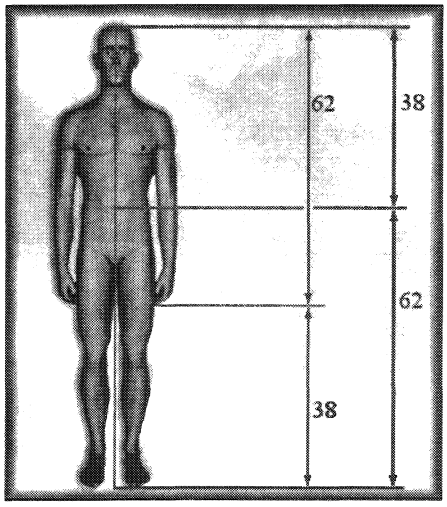
Ancient scientists were surprised to discover the mathematical connection between the main cycles of the solar system and one of the "Platonic Solids", the dodecahedron. The Egyptians ranked the dodecahedron as the "main Universe figure," symbolizing the "Harmony of the Universe." They thus based their systems of measurement (calendar, time and angle measurement) on the numerical parameters of the dodecahedron (12 faces, 30 edges, 60 plane angles on the dodecahedron surface). These systems were coordinated with their "Theory of Harmony" based on the Golden Proportion which underlies the dodecahedron. According to this hypothesis, mankind has lived for thousands of years according to the Golden Section!

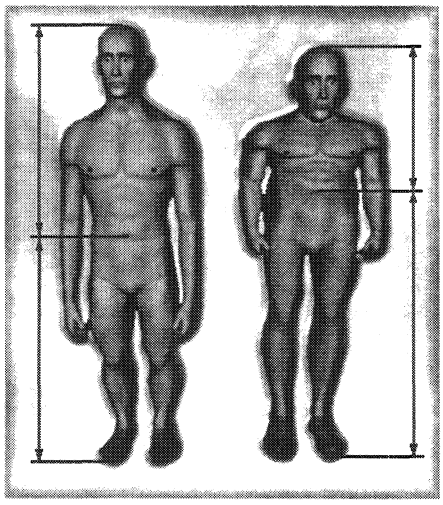
4. Golden Section, Nature, and Man

This hall consists of two exhibitions: (1) Golden Section in Nature, and (2) Golden Section and Man. In the former exhibition, numerous applications of the golden section (pentagonal symmetry, golden spirals, and Fibonacci numbers, for example) are given. In the latter, examples of paintings and sculpture are used to illustrate the golden section as a formula for beauty.

Let us consider some exhibitions. In 1912 in El-Amarn, Egypt, the workshop of the Egyptian sculptor Tutmes was excavated. Contents of the workshop included the sculptural portrait of the ancient Egyptian queen Nefertiti (meaning "The beautiful woman is coming"), which is considered symbolic of perfect beauty in a woman's face. In 1974 the Russian artist Jury Raksha created the picture "Harmony," whose facial beauty does not surpass that of Nefertiti.

It is well known that a harmonic man's body is divided by the navel into the Golden Section (Figure (a) below). Men's bodies that do not fit the Golden Section are shown in Figure (b).





(a) (b)

Harmony (a) and disharmony (b) of man's body

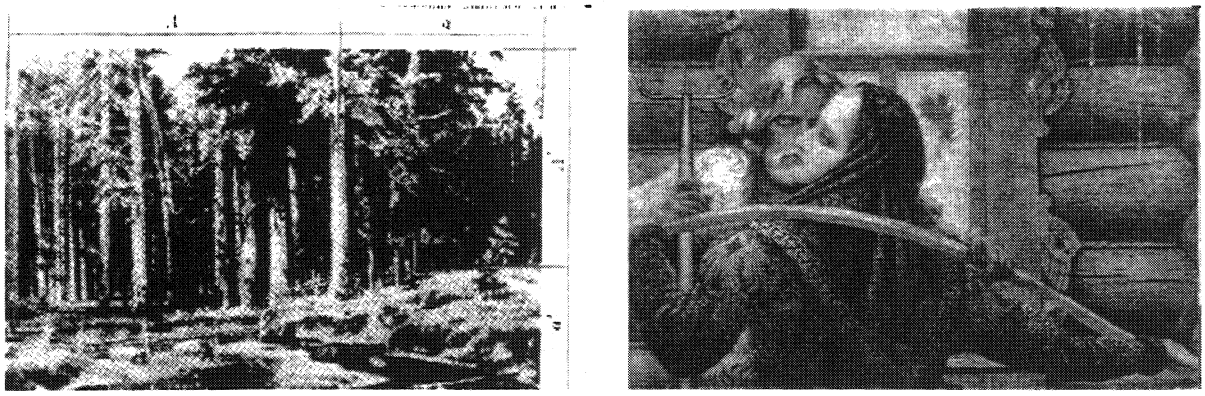
5. Golden Section in Art (Music and Painting)

This hall includes the brightest examples of the Golden Section in music and visual art. Let us consider the most interesting of them.

5.1. Chopin's Etudes in Lighting of the Golden Section. Any musical composition has a temporary duration and is divided into separate parts by some "aesthetic stakes", which facilitate our perception of the piece. The Russian musicologist Sabaneev in his article [3] shows that the separate time intervals of the musical pieces connected by the "culmination event" are, as a rule, in the ratio of the golden section. In Sabaneev's opinion, the quantity and frequency of the golden section usage in musical composition depends on the genius of the composer. Those musical pieces distinguished by the most frequent use of the golden section come from the most brilliant composers, that is the intuition of the form and ordering, as it is necessary to expect, is strongest for the first class geniuses.

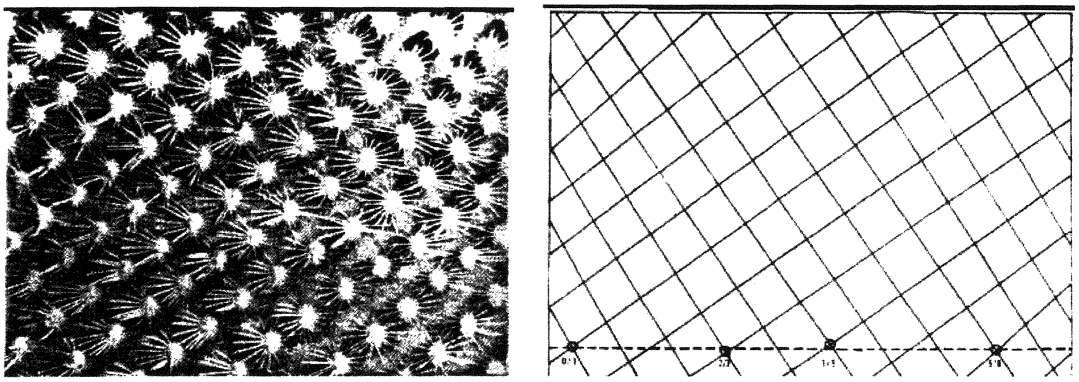
According to Sabaneev's observations, the greatest number of musical pieces based on the golden section are observed in works of Arensky (95%), Beethoven (97%), Gaidn (97%), Mozart (91%), Scriabin (90%), Chopin (92%), Schubert (91%). Chopin's etudes were studied by Sabaneev in detail. He found that 154 exhibited the Golden Section; only in three etudes was it missed. In some cases the construction of the musical piece combined symmetry and Golden Section simultaneously; in these cases the piece was divided in some symmetrical way, and the Golden Section was observed in each. For example, many of Beethoven's compositions are divided into two symmetrical parts, and the Golden Section is observed in each part.

5.2. Golden Section in Painting. Investigating the compositional structures of paintings, the masterpieces of world art, critics have observed that the Golden Section is widely used in landscape painting.



Usage of the Golden Section in Shishkin's painting "The Ship Grove" and Konstantin Vasil'ev's painting "Near to the window"

5.3. Phyllotaxis Lattices. In the world of botany, Fibonacci numbers and the Golden Section may be seen in "phylotaxis" or growth patterns of plants. For example, cactus areoles (concentrations of thorns) are placed in spirals, and the numbers of left-hand and right-hand spirals are the consecutive Fibonacci numbers 21 and 34. If presented on a plane surface the following raster lattice may be observed in which there are 21 lines with right-hand slope and 34 lines with left-hand slope.



Cactus surface and its geometrical model

The Austrian scientist Paturi, author of the remarkable book "The Plants as Ingenious Engineers of Nature" [4], found the use of raster lattices in the paintings of great Renaissance artists; in particular, Titian's painting "Vakch and Ariadna". He wrote: "In all times the artists, consciously or unconsciously, studied to comprehend the laws of aesthetic perception by watching nature. The artists were enchanted always by the simple and simultaneously rational geometry of the biological growth forms." (the quotation is taken from [4], p. 84-85; English translation was made by A. Stakhov).

6. The halls "Mathematics of Harmony", "Fibonacci Computers", and "Fibonaccization of Modern Science"

These halls contain exhibitions that are very important from the scientific point of view. Concepts of the Harmony Mathematics, Fibonacci computers and new coding and cryptography theory based on the Fibonacci matrices are stated in [7-12].We will consider only one exhibition from the hall "Fibonaccization of Modern Science".

Fibonacci Numbers and the Genetic Code. In 1990 Jean-Clode Perez, an employee of IBM, made a rather unexpected discovery in the field of genetic code [5]. He discovered the mathematical law controlling the self-organizing of the T, C, A, and G bases inside of DNA. He found that the consecutive sets of DNA nucleotides are organized in frames of the distant order called "Resonances." Here "Resonance" represents the special proportion ensuring division of DNA parts pursuant to the three neighboring Fibonacci numbers (1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...), for example 55-34-21, 89-55-34, etc.

Let's consider the DNA molecule of insulin, one of the simplest DNA molecules. It consists of two circuits, α- and β-circuits. For the β-circuit the sequence of triplets has the following form:

ATG-TTG-GTC-AAT-CAG-CAC-CTT-TGT-GGT-TCT-CAC-CTC-GTT-GAA-GCT-TTG-TAC-CTT-GTT-TGC-GGT-GAA-CGT-GGT-TTC-TTC-TAC-ACT-CCT-AAG-ACT

If we mark all T-bases in red and the others by yellow and count the number of all bases (90), we have 34 T-bases and 56 other bases. Thus we have the following proportion between the bases: 90-56-34, very close to Fibonacci's "resonance": 89-55-34. It means that Jean-Clode Perez's law is fulfilled for the insulin DNA molecule with sufficient accuracy for practice.

This discovery is doubtless an outstanding one in determining the development of gene engineering. In the opinion of Jean-Clode Perez, the SUPRA-code of DNA indicates the highest level of self-organizing of nucleotides in DNA according to the principle of the golden section. This surprising discovery allows a further connection between music, poetry, market fluctuations (Elliott Wave theory) and genetic code. It is clear that the harmony of Chopin's etudes, Pushkin's poetry, and Elliott's waves are similar to the harmony of the genetic code, in which Fibonacci's resonances are seen both in the DNA molecule and in all its parts.

This and other modern scientific discoveries based on the Golden Section (Shechtman's quasi-crystals, Bodnar's theory of phyllotaxis, Soroko's Law of structural harmony of systems, resonance theory of the Solar system, Elliott Waves) show that man is coming close to uncovering one of the most complicated scientific notions, the notion of Harmony, which Pythagoras believed underlies the Universe.

7. Harmonic Education

It is clear that awareness of the Golden Section and its applications in Nature, Music, Art, and Science could enrich the life and work of scholars in a variety of professions. A question arises instinctively: Why is this information not taught in secondary school? Educators seem unable to answer this question. And frankly speaking, we, the authors of this paper, cannot answer this question either.

The reason for this may be tradition. Traditionally the classic pedagogy treats the Golden Section with some prejudice, as if it were related to astrology and the esoteric sciences. But the connections we have enumerated here—phyllotaxis, Shechtman's quasi-crystals, Perez's discovery, and others—show that the classic "materialistic" science is now moving to embrace the "esoteric" science!

The astronomer Johannes Kepler once said: "There are two treasures in Geometry: Pythagorean theorem and division of a line segment in extreme and mean ratio ("golden ratio"). The former can be compared to value of gold; the latter can be named as a gemstone." But every pupil knows the Pythagorean theorem; sometimes even famous scientists and teachers have a dim knowledge of the Golden Section. The first step in improving education is integrating the Golden Section into the school mathematics (geometry, algebra, number theory); the second step is to integrate it into the natural and social science disciplines (physics, chemistry, astronomy, botany, biology, anatomy, psychology, sociology, economics, and computer science). We should also teach the principles of the Golden Section in art classes (golden rectangle, golden spiral, pentagon, Fibonacci lattices, etc.) and include examples of their usage in architecture, painting, sculpture, music, and poetry. Thus our teaching would promote a new scientific outlook based on principles of harmony and the golden section.

8. Architectural Realization of the Museum of Harmony and Golden Section

The authors' main goal is the physical realization of the Museum of Harmony and the Golden Section as a grandiose architectural temple in which works of nature, science, and art based on the Golden Section would be collected. We are therefore asking outstanding architects to participate in this project, and we ask businessmen, creative persons, sports figures, artists, actors, musicians, and anyone else who could support the project financially to become sponsors and participants in this undertaking. In the meantime, we welcome our readers to visit our "Virtual Museum of Harmony and Golden Section" at http://www.goldenmuseum.com/ [6].

References

[1] Luca Pacioli. *Treatise on Counting and Records.* Moscow, Publisher "Finance and Statistics", 1983 [In Russian].
[2] I.P. Shmelev. *Phenomenon of the Ancient Egypt.* Minsk, Publisher "Lotaz", 1993 [In Russian].
[3] Z. Sabaneev. "Shopin's Etudes in the lighting of the golden section". *Art Journal "Iskusstvo",* Volume 2, 1925, pp. 132-135 [In Russian].
[4] F.R. Paturi. *The Plants as Ingenious Engineers of Nature.* Moscow, Publisher "Progress", 1982 [In Russian].
[5] Perez, Jean-Clode. WEB site "The DNA SUPRA code", 2000. http://www.genum.com/dna_supracode/dna_supracode.htm .
[6] A.P. Stakhov and A. A. Sluchenkova. Web site "Museum of Harmony and Golden Section", 2001 http://www.goldenmuseum.com/
[7] A.P. Stakhov. *Computer Arithmetic based on Fibonacci Numbers and Golden Section: New Information and Arithmetic Computer Foundations*, Toronto, SKILLSET Training, 1997.
[8] A.P. Stakhov. "The Golden Section and Modern Harmony Mathematics". – *Applications of Fibonacci Numbers*, 1998, V. 7, 323-399.
[9] A.P. Stakhov "The Golden Section and Modern Harmony Mathematics". – *Boletin de Informatica.* The Golden Section: Theory and Applications, 1999, No 9-10, 3-24.
[10] A.P. Stakhov. A generalization of the Fibonacci Q-matrix. *Reports of the National Academy of Sciences of Ukraine,* 1999, No 9.
[11] A.P. Stakhov, V. Massingua, A.A. Sluchenkova. *Introduction into Fibonacci Coding and Cryptography.* Kharkov: Publisher "Osnova" of the Kharkov State University, 1999.
[12] A.P. Stakhov. "Brousentsov's Ternary Principle, Bergman's Number System and Ternary Mirror-symmetrical Arithmetic". *The Computer Journal,* Vol. 45, No 2, 2002, 221-236.