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Exploring the Effect of Direction on Vector-Based Fractals

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Abstract

This paper investigates an approach to begin the study of fractals in architectural design. Vector-based fractals are studied to determine if the modification of vector direction in either the generator or the initiator will develop alternate fractal forms. The Koch Snowflake is used as the demonstrating fractal.

Introduction

A fractal is an object or quantity that displays self-similarity on all scales. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales [7]. Fractals were first discussed by Mandelbrot in the 1970s [4], but the idea was identified as early as 1925. Fractals have been investigated for their visual qualities as art, their relationship to explain natural processes, music, medicine, and in mathematics [5].

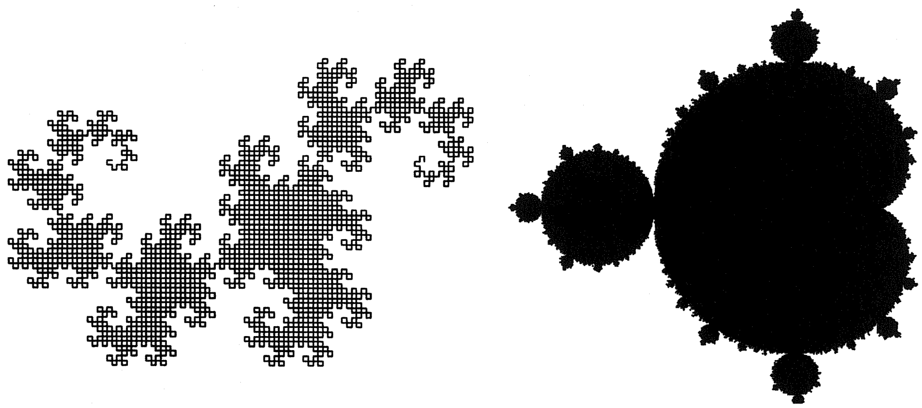
Javier Barrallo classified fractals [2] into six main groups depending on their type:

1. Fractals derived from standard geometry by using iterative transformations on an initial common figure.
2. IFS (Iterated Function Systems), this is a type of fractal introduced by Michael Barnsley.
3. Strange attractors.
4. Plasma fractals. Created with techniques like fractional Brownian motion.
5. L-Systems, also called Lindenmayer systems, were not invented to create fractals but to model cellular growth and interactions.
6. Fractals created by the iteration of complex polynomials.

From the mathematical point of view, we can classify fractals into three major categories. The first, IFS, iterated function system, like Koch Snowflake, Cantor set, Barnsley's Fern and the Dragon Curve shown in Figure 4. This method can generate a fractal from any set of vectors or any defined curve. The second is the complex number fractals. They can be two-dimensional, three-dimensional, or multiple-dimensional. They represent a single case of the IFS that is using the complex numbers or the hyper complex numbers in a Cartesian plane to plot the fractals. The Mandelbrot set and Julia set are examples of these. The third is orbit fractals. They are generated by plotting an orbit in two or three-dimensional space. Examples include the Bifurcation orbit, Lorenz Attractors, Rossler Attractors, Henon Attractors, Pickover Attractors, Gingerbreadman, and Martin Attractors. These are associated with chaos theory.

By examining these fractals, we were also able to classify them into two major categories depending on the method they could be created, or the mathematical method used to calculate them.

From the drawing method point of view, the first method is a line or vector fractal. These are generated from the recursive replacement of vectors, like the Dragon Curve in Figure 1a. The second are fractals that are generated as a group of points in the complex plane, such as the Mandelbrot set and the Julia set, as in Figure 1b. Some fractals that exhibit a vector quality can also be generated by point plotting methods.



a. Dragon curve b. Mandelbrot set

Figure 1. *Fractal types*

In this research, we are interested in vector-based fractals and the use of the replacement concept and the iterated function system as a way to generate them. These fractals have directional and geometrical properties that make them possibly suitable for applications in architecture. Vector-based fractals can be described in terms of vertices and the lines connecting them. This has the potential to be used directly as architectural elements or to simply use the vertices to define the locations of such elements.

Chris Yessios with Peter Eisenman [8] were among the first to write about utilizing fractals and fractal geometry in architecture. Yessios described a way computers can be introduced to architectural design as an explorer and generator of architectural forms. He used the fractal geometry, arabesque ornamentation and DNA/RNA biological processes as fractal generators. A fractal program was developed that enabled him to use several generators on the same base and to go many steps forward in the iteration process, as well as, backward. The project that was used in this investigation was a studio project to design a building for a competition given a specific architectural program.

More recently, S. Durmisevic and O. Ciftcioglu [3] discussed another approach on how fractals can be used in architectural design. They proposed to use a fractal tree form as an indicator of a road infrastructure and another fractal to determine the type of architectural forms to place along this transportation spine. Fractal geometry was used for architectural forms and urban planning.

No other similar studies could be found where fractals were used as architectural form generators to suggest three-dimensional forms. The results from these two suggest that the explicit self-similarity and the repetition of the same shape may distract from developing interesting architectural forms, so both developed means to modify the replication.

For this particular study we intend to investigate only the directional property found in generating vector-based fractals.

Creating a fractal

A vector-base fractal, Figure 2, is composed of two parts: the *initiator* and the *generator*. For example, the Koch Snowflake starts with an equilateral triangle as the initiator. The generator is a line that is divided into three equal segments. The middle segment forms an equilateral triangle.

By replacing every line of the initiator with the full generator, we get the first iteration of the snowflake. By iterating this operation again and again, replacing every line of the new initiator with the full generator, we end with a figure that approximates a snowflake. The iteration process can continue to infinity to generate a real Koch Snowflake fractal, but as we are interested in the evolving form, we only iterate the function for some finite number of times. Figure 3a displays the Koch Snowflake for each of three iterations. Normally, an alternative version can be created if the generator is changed, inverted. It then becomes the Koch Antisnowflake, Figure 3b.

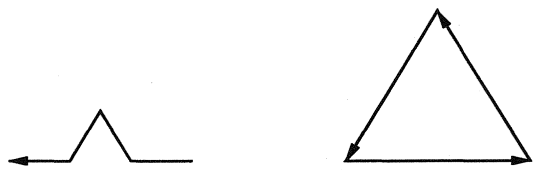


Figure 2. *Generator and initiator*

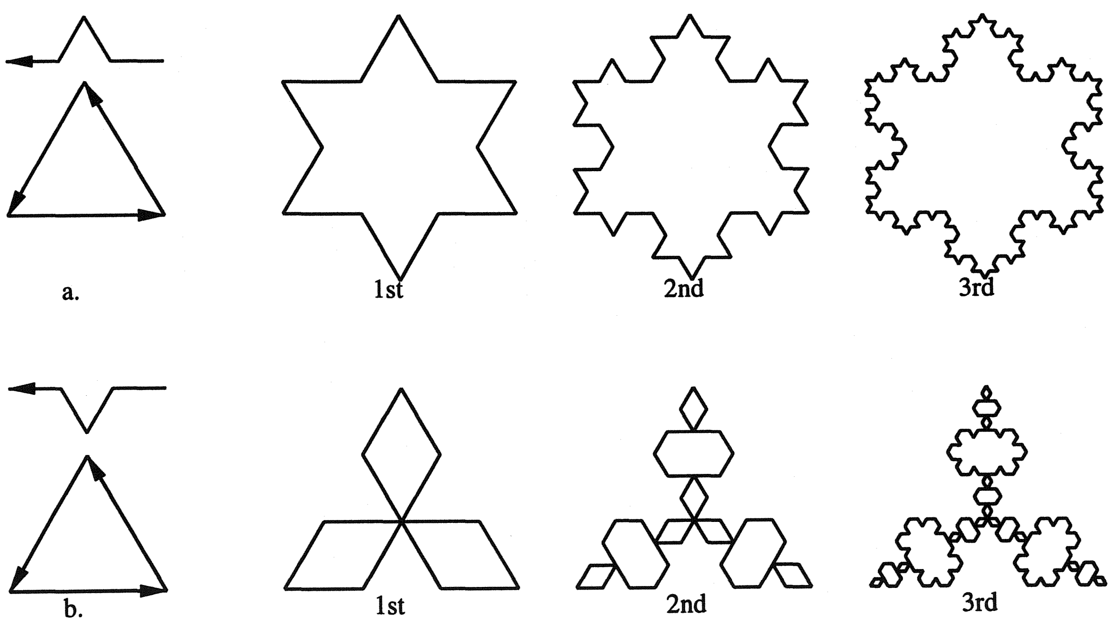


Figure 3. The Koch Snowflake and the Antisnowflake Shape of generators and initiators

There is a group of fractals, including Mandelbrot set variations that have been formally identified in his book "The Fractal Geometry of Nature" [4] as depending on the concept of replacement. Some of the IFS fractals are: Cantor Set, Barnsley's Fern, Koch Antisnowflake, Koch Snowflake, Box Fractal, Cantor Square Fractal, Cesàro Fractal, Dragon Curve, Gosper Island Fractal, H-Fractal, Sierpinski Curve, Minkowski Sausage. Some of these are displayed in Figure 4.

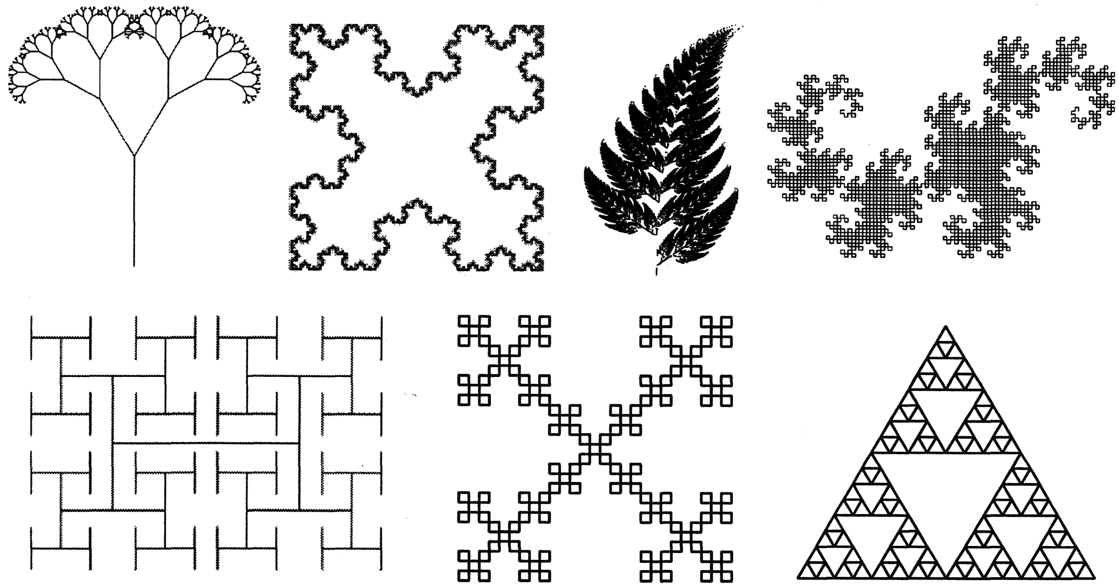


Figure 4. *Tree fractal, Cesàro fractal, Barnsley's Fern, Dragon Curve, H-fractal, Sierpinski curve, square and triangle.*

All of these fractals are based on simple geometric shapes. Shakiban and Berstedt [6] discussed a new generating procedure based on vector calculus and modular arithmetic to generate the Koch Snowflake. The procedure was then applied to create more generalized snowflakes rather than the triangular classical snowflakes. They also suggested the use of n-sided polygons, such as pentagons as initiators. In all cases the replacements made at each iteration are consistent and unvaried.

Another interesting method to modify the geometric shapes produced by a fractal comes from the selection of the direction and displacement of the initiator. Mandelbrot discussed this when he was describing the random Koch coastline, and Brownian fractals and the random midpoint displacement curves.

Direction

As we started to investigate the selection of the generator and initiator, our initial focus was on the effect of direction of the initiator on the produced fractal as related to the generator. Normally, the length of the generator is equal to the length of a segment of the initiator and the direction of the line segments in the generator and initiator are the same as seen in the Koch Snowflake in Figure 3.

One possible variation is to modify the direction of the entire generator or initiator or individual parts of each and the orientation of the generator as it is placed on the initiator.

Figure 5 displays the Koch Snowflake with the generator in one direction and the normal and reversed direction for the initiator. When the initiator is reversed the Antisnowflake appears. Figure 6 reverses the direction of the placement of the generator. The fractals developed starting at the second iteration are mostly undocumented versions of the Koch Snowflake. Manuel Alfonseca and Alfonso Ortega described one such variation while demonstrating using L-system to generate fractals [1].

Additional variations were found by modifying the direction of individual line segments within the generator or the initiator, as shown on Figure 7.

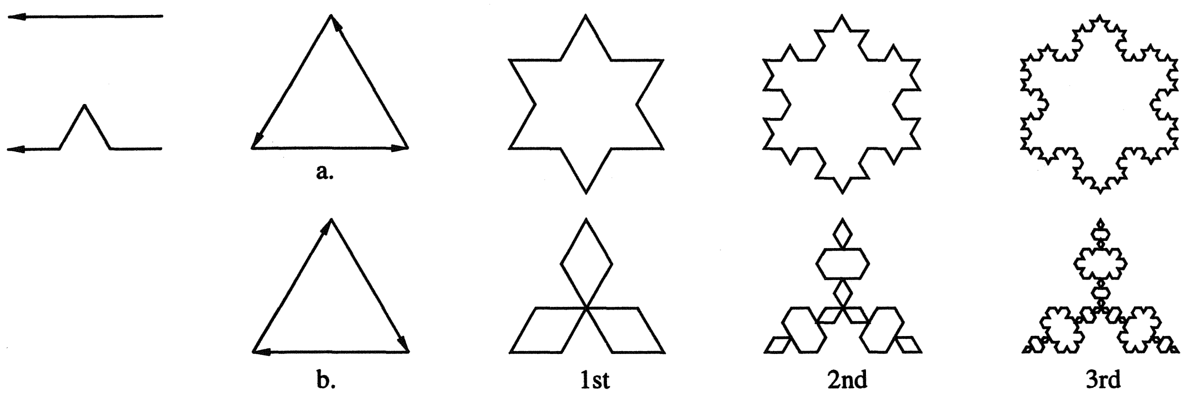


Figure 5. *The direction effect on the generated fractal applied on the simple Koch Snowflake.*

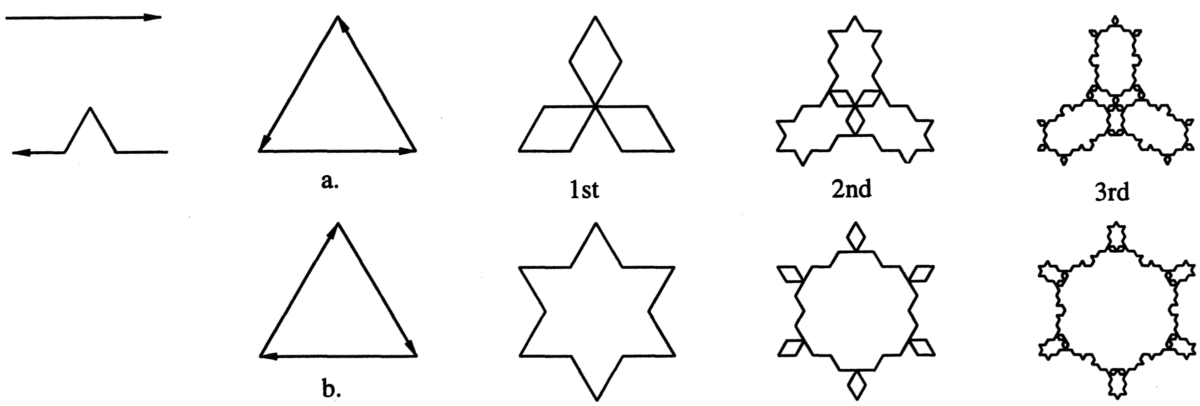


Figure 6. *The effect of the direction of the placement of the generator on the generated fractal applied on the simple Koch Snowflake.*

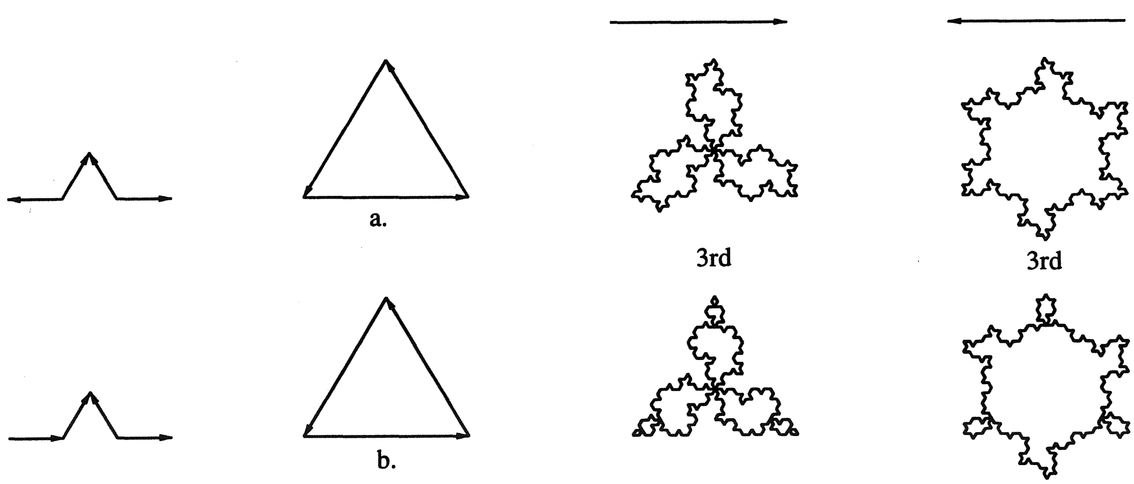


Figure 7. *The effect of the generator with different directions of its components after three iterations.*

Another variation was explored by making the generator more complex, still using the original vectors. The generator in Figure 8 is two sets of the same vectors going in two different directions, one is from left to right and the other set is from right to left. This configuration shows all the possibilities of alternating the direction of replacement. Figure 9 is a double set of vectors using the classical Koch and anti-version generator combined. Changing the direction of the vectors on the initiator produces the same results. In this case, the fractal in Figure 9 can also be constructed by overlaying fractals in Figures 8a and 8b.

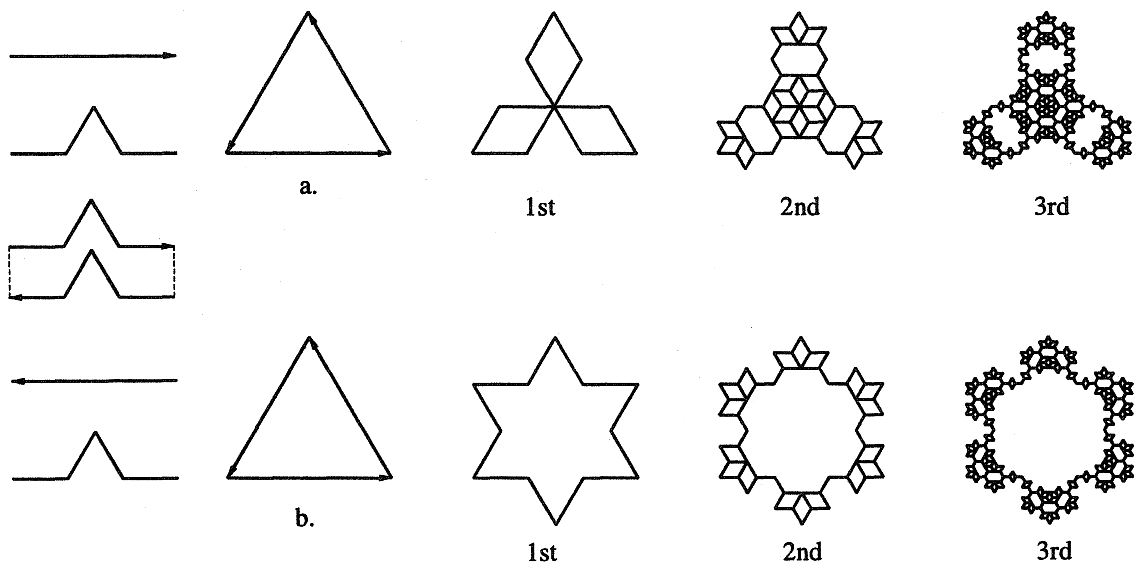


Figure 8. *The effect of double set of the same vectors, which have different directions.*

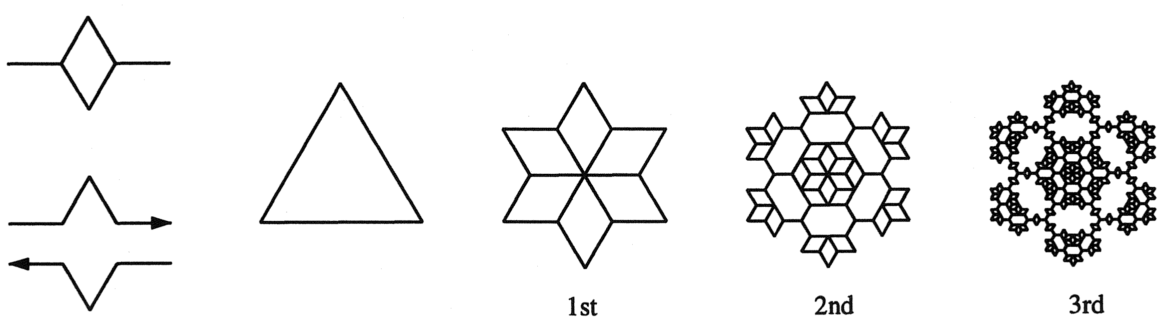


Figure 9. *A general Snowflake with all the possibilities of directions is created through the application of a generalized generator where the vectors go opposite directions.*

Observations

Investigating the directional effect of the generator can lead to some unidentified fractals with different visual characteristics.

The Koch Snowflake was further extended to a square initiator, creating the Cesàro fractal. A similar series of explorations were also conducted with the Lévy fractal, also known as C-Curve, the Sierpinski Curve, and another version of the Koch, the Exterior Snowflake fractal. All give similarly interesting results. We concluded that within classical named fractals, these are other fractals that can be found using the original generator and initiator but modifying their direction. These experiments offer us further opportunities to study fractals and determine if their geometry can be useful in generating architectural forms.

These alternative methods of interpreting the generator and the initiator can also be used by a teacher who wishes to show some of the hidden visual gems that can be found by investigating variations that still use the same basic forms. This offers students a starting point for considering how to generate their own fractals. Simple modifications can readily lead to sometimes unexpectedly interesting results.

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