BRIDGES
Mathematical Connections
in Art, Music, and Science

Vasarely Design and Other Non-Cubical Designs

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Abstract

I exhibited 3-dimensional designs made from Rubik's cubes at the Bridges Conference two years ago. I left the conference with new ideas brought forth by the conference participants. Ken Landry [1] suggested a block model for a design that I named after him, the Landry Staircase (figure 3). This design ushered in a new type of non-cubical designs, the so-called block designs. Craig Kaplan [2] gave me an idea for a simple but rather nice cubical design. A call for a "practical application" of those designs prompted a response: use the designs as chandeliers, placing small candles at strategic points on the design, creating various arrays of lights. In complete darkness those lights form a specific configuration "suspended in air." I am assuming that it is so dark the designs cannot be seen; only the lights penetrate the darkness.

Two years have passed since the conference and I still cannot find anyone, on or off the web, who creates those designs. People at the conference complained about the "complexity" of the design problem and its solution and suggested that a computer be used to implement those designs. But how to do it? I am not a computer programmer. I solved the problem purely analytically. All you need to do is use existing mathematics and develop some rules to simplify the solution and implement those designs.

After reviewing a few basic concepts, I will show you how to construct Vasarely, my oldest non-cubical design. I encourage you to go through the process of twiddling all 102 Rubik's cubes. After that, I will discuss other non-cubical designs. By all means, build the Landry Staircase if you can get the required 216 Rubik's cubes.

Review of Design Basics

A Rubik's cube has six unique colors on its six faces. Realizing this, David Singmaster devised a notation [3] for faces and rotations that depends upon the position of each face on a fixed cube with F for front face, B for back face, R for right face, L for left face, U for up face and D for down face. A rotation of the up face, say, is labeled as U for a clockwise rotation by 90 degrees, U2 for a rotation by 180 degrees and U' for a clockwise (counterclockwise) rotation by 270 (90) degrees. One may consult a number of books, including Ref. [4]. The Singmaster notation has become a standard tool in describing cube solution algorithms.

A Rubik's cube is made up of pieces called cubies. Corner cubies are pieces that form corners of the Rubik's cube. Edge cubies are pieces that form edges of the Rubik's cube. Center cubies are pieces that form centers of the Rubik's cube. In a 3x3x3 Rubik's cube, there are 8 corner cubies, 12 edge cubies and 6 center cubies. Individual Rubik's cubes form pieces of a design. In a design, there are corner cubes, edge cubes and center cubes. Corner cubes are cubes whose three faces participate in a design. This general definition includes cubes in non-cubical designs, where the conventional meaning of a corner no longer makes sense. Edge cubes are cubes that act as edges of a given design and contribute two faces to a given design. Center cubes are cubes that act as centers of a given design and contribute only one face.

Parity pair is purely a design concept that is not well known. Suppose you have two cubes of identical colors on opposite faces (a so-called color scheme). Let the colors on a pair of opposite faces of one cube be switched relative to the other cube. Such a duo of cubes is called a parity pair. One can bring such a pair together in a two-cube structure in which the faces that touch are colored the same. Therefore, this two-cube structure will have five colors only on its combined six faces. The sixth color is hidden inside this structure or suppressed. Bringing two such two-cube structures together will form a four-cube structure with four colors only on its combined six faces. Two colors are suppressed. Bringing two such four-cube structures will produce an eight-cube clean "design" with three colors only on its combined six faces. The other three colors are suppressed. To gain a better understanding of this important design concept, one is encouraged to obtain eight cubes in four parity pairs and construct such a design.

Putting those cubes as corners of an arbitrary cubical or rectangular-solid design produces reflection-invariant designs and ushers in a concept of parity-pair induced design symmetry. Reflection invariance is the simplest design symmetry due to parity pairs. A reflection-invariant design is one whose opposite faces are identical both in color and geometry. It is clear that, in such a design, the color of the center cubie of the left face of a corner cube must be identical to the color of the center cubie of the right face of its neighboring corner. The same is true of the colors of the corner cube's up/down and front/back faces. This can only be achieved if the corner cube forms parity pairs with each of its neighboring corners. Other requirements on parity pairs of the edges are possible, but I will not pursue this topic further. See Marie and Jaroslav designs [5] and Čtyřspřeží Design [6].

Vasarely Design

Vasarely (figure.1) is a non-cubical design. This means the design is not a pure cube. One cuts a 3x3x3 "hole" into a purely cubical 5x5x5 design. One places four cubes into this hole as shown. The design displays three colors in a checkerboard pattern. The faces that cannot be seen also display three colors as well as the checkerboard pattern.

We may think of the design as consisting of five layers. The first layer touches the table, upon which the design stands and consists of a "square" of 25 cubes. The second layer forms a "sandwich" with the first. Note the continuity of the checkerboard pattern. This continuity must prevail on the whole design. The third, fourth, and fifth layers consist of 16 cubes each, placed on top of each other. They truncate what would normally be three layers of a 5x5x5 checkerboard design, producing the above-mentioned hole.

To construct a checkerboard pattern on a corner cube

1. Hold cube fixed so that its front face is blue, right face is red, and up face is green.
2. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
3. Turn cube upside down so that its front face is red, right face is blue, and up face is yellow.
4. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
5. Hold cube fixed so that its front face is red, right face is green, and up face is blue.
6. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
7. Turn cube upside down so that its front face is green, right face is red, and up face is white.
8. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
9. Hold cube fixed so that the front face is green, right face is blue, and up face is red.

10. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
11. Turn cube upside down so that its front face is blue, right face is green, and up face is orange.
12. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
13. Hold cube fixed so that its front face is blue, right face is red, and up face is green.
14. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.
15. Turn cube upside down so that its front face is red, right face is blue, and up face is yellow.
16. Do L2R2U'L2R2D'RL'F2L2R2B2RL'D2.

The patterned cube above would go as a bottom-layer corner in the photograph. Its neighboring corner would undergo exactly the same set of operations, except done on a parity pair. This is a general feature of a checkerboard design with an odd number of Rubik's cubes on a side. For a discussion of this topic see Reference [4].

We notice that the sequence of twiddles is repeated. This is not an error. If multiple application of the above strategy would be done on a fixed cube, then the cube would become solved after a third application of this strategy. But the cube is turned such that its faces display different colors so the strategy works on different cubies, gradually building the pattern on an individual cube piece by piece.

The steps above construct a checkerboard on a single cube. Care must be taken to make sure the checkerboard shows continuity from cube to cube. If a face of one cube has green-blue checkerboard with blue center cubie, then its next-door neighboring cube will have a green center cubie on the blue-green face. Also, don't forget parity pairs.

The first one to recognize Vasarely was Argonne National Laboratory (ANL). This design was one of ten winners in a juried exhibit, sponsored by Arts at Argonne in 1997. The details may be found at http://www.anl.gov/OPA/sciart. Clicking on the picture will enlarge it. Just last year this website was spotted by Nick Mee [7], a founder and CEO of an English company specializing in educational software called Virtual Image. He is participating in a project called "Connections in Space" that uses Vasarely design in their website. The URL to view is http://www.connectspace.co.uk. This URL deals with connections in arts, mathematics and science. I highly recommend visiting this site.

Block-Model Designs: the Landry Staircase

At the Bridges Conference in 2000, I met Ken Landry who was exhibiting polyhedra. One of his creations caught my eye. It was a 27-cube structure. One of the corners together with its immediate neighboring edges were inverted and placed upon the truncated upper face as shown in figure 2. Figure 2 is not a design, but a block model of one. The cubes used could be any cubes. The reason I use Rubik's is their immediate availability. The actual design is constructed by replacing each model cube by an eight-cube design in four parity pairs. Exactly how patterns on individual cubes of this mini design are constructed is beyond the scope of this article. An interested reader is asked to consult Reference [4]. You need to replace each model cube by eight cubes so the design uses 216 Rubik's cubes. The result can be seen in figure 3a.

Figure 3b shows the Landry Staircase as a candle holder. Place a tealight candle in a small cup (to protect the cubes) upon each of the nine "steps." You will get nine tealights in a rhombic arrangement. The idea of using selected non-cubical designs as candle holders came from 2000 Conference participants. They wanted practical applications for those designs, and this is the only "practical" application I conceived. I also found that larger block cubes could be used in this way, generating models for larger staircases. Then one could place more candles on such staircases, one candle per step. I would like to construct a Landry Staircase whose block model would be 125-cube model, in a 5x5x5 arrangement. Unfortunately, such a design calls for a thousand Rubik's cubes, and I do not have that many.

Another block model design is Skřivánek design, shown in figure 4. Although I do not have the block model, it can easily be visualized. Other block model designs are of course possible. A Rubik's cube designer should keep his or her eyes open for nice block model designs.

Other Designs

The Kaplan Kube, shown in figure 5, is based upon a picture shown by Craig Kaplan during his lecture at the Bridges conference in 2000. The picture was an impetus for a rather simple 5-color cubical design. Perhaps some readers will appreciate it. I have to sound a warning to those who would like to see their work captured in a Rubik's cube design: it is not always possible! The medium, that is, Rubik's cubes, is very restrictive. I cannot work that medium into spheres, spirals and such structures. I may admire someone's work, but it might be impossible to create a Rubik's cube design based on it.

The other non-cubical designs are shown in figures 6 through 8. Observe that figures 3 and 4 show block-model designs of different shapes but identical basic mini designs. If you have created the Landry Staircase as a block design, you don't have to do any twiddling to construct Skřivánek design. Just remove unneeded mini designs and leave only the needed ones. Figures 6 and 7 show two designs of identical shapes but different patterns. Those who have successfully implemented Design for Pedestrians, discussed in Ref. [6], will find the design in figure 6 particularly familiar. The design in figure 8, called Petrášek design, is merely a 2x2x2 cube standing upon a 4x4x2 rectangular solid. The overall design of the cube is identical to the design in figure 7. The possibilities and variations of non-cubical designs are endless.

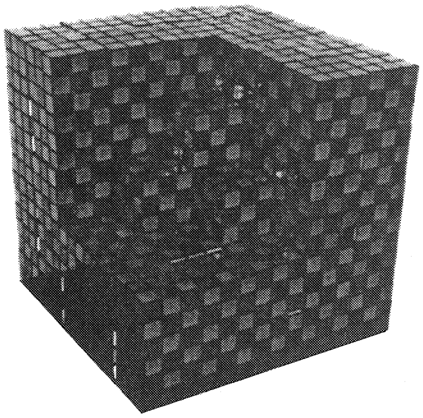

Figure 1: *Vasarely Design*

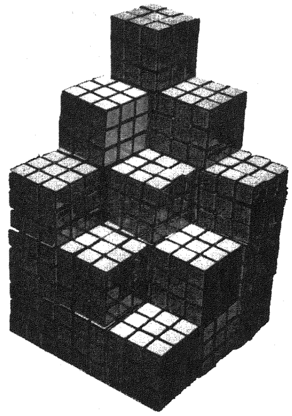

Figure 2: *Block Model for Landry Staircase*

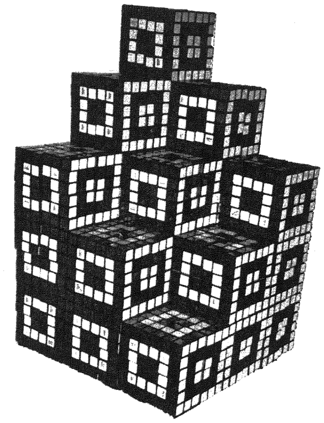

Figure 3a: *Landry Staircase*

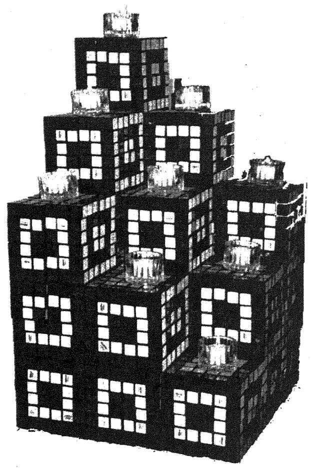

Figure 3b: *Landry Staircase as a Candle Holder*

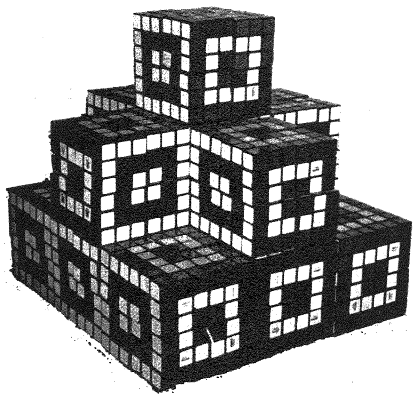

Figure 4: *Skřivánek Design*

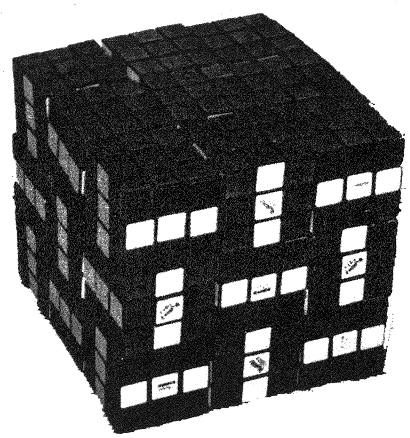

Figure 5: *Kaplan Kube Design*

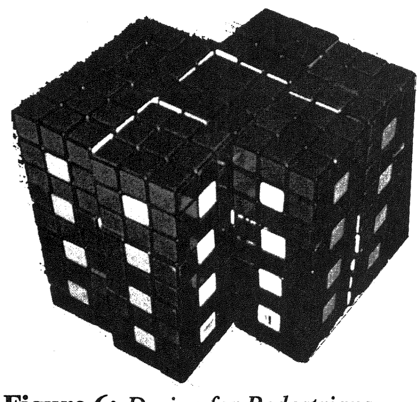

Figure 6: *Design for Pedestrians*

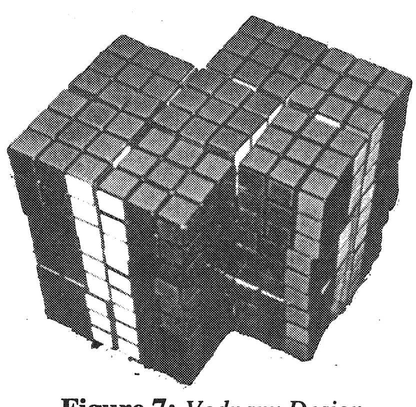

Figure 7: *Vodnany Design*

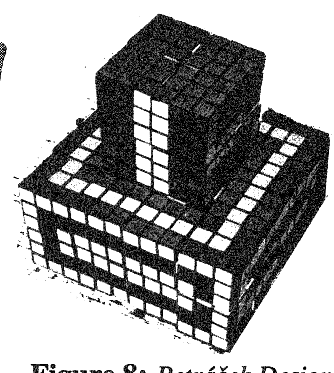

Figure 8: *Petrášek Design*

References.

[1] Ken Landry, private communication
[2] Craig Kaplan, private communication
[3] D. Singmaster, *Notes on Rubik's Magic Cube*, Enslow Publishers, 1981
[4] Hana M. Bizek, *Mathematics of the Rubik's Cube Design*, Dorrance, 1997
[5] Hana M. Bizek, *The Rubik's Cube Design Problem*, Bridges Conference, 2000
[6] Hana M. Bizek, *Designs from Rubik's Cubes*, MOSAIC Conference, 2000
[7] Nick Mee, private communication