BRIDGES
Mathematical Connections
in Art, Music, and Science

Study and Application of African Designs for Use in Secondary Education

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Abstract

This article suggests ways in which the artwork of the African people can be used to introduce certain geometric concepts to secondary school students. Some examples for the symmetry groups for one-dimensional patterns in African artwork are presented. Further suggestions are provided on construction of the regular polygons that lead to some of the motifs found in African designs. Suggestions are made for the use of some design pieces to prove geometry theorems

1. Introduction

The continent of Africa has tremendous cultural, linguistic, ethnic, and artistic diversity [5]. As one travels down from North Africa to its southern most tip, one can find expression of cultural heritage and beliefs in the artwork of the Africans, despite economic, political, and social hardships. "Traditional artistic practice across the continent continues to retain a vitality and social relevance" [10]. Studying the mathematics of these cultures offers a unique opportunity for students to "experience multicultural mathematics activities that reflect knowledge and behavior of people from diverse cultural environments" [1]. By studying the designs on pieces of pottery, in woven cloth, and in the artwork done by many African women to adorn the walls of the family home, students can become familiar with concepts such as symmetry, transformation, frieze patterns, and the construction of geometric figures using a compass and straight edge only. In doing this, students learn "not only to value mathematics but, just as important, they may develop a greater respect for those who are different from themselves" [1].

Ways are suggested in which various pieces of African art can be used to illustrate geometric concepts that fall within the secondary school mathematics curriculum. The constructions of various designs are analyzed and suggestions are given as to how to incorporate these constructions into a classroom lesson. Various software programs are mentioned that may add value and interest to these geometry lessons.

The paper starts with some examples of African art that illustrate the seven frieze groups. The symmetry in some finite designs is discussed and constructions for the creation of these designs are suggested. Some thought is given to using pieces of African art to prove geometry theorems.

2. Seven Friezes in African Art

A study of designs in African artwork affords an opportunity for students to identify the seven frieze groups [7]. Students can be encouraged to study each design and then identify the relevant frieze group. As a follow-up to this activity, students can be encouraged to use software such as Geometer's Sketchpad and Java Kali [11] to create frieze patterns of their own. Java Kali is a software program that allows students to experiment with their own frieze patterns and create a wide variety of designs. As a cross-curricular activity, students could be encouraged to create a piece of pottery or ceramic work in an art class. They could be encouraged to decorate their creation with a frieze pattern of their own design. The following are some examples of frieze patterns in African artwork.

The Ashanti in Ghana are well known for their multi-colored woven cloth. An example of one of their designs in figure 2(a) has translational symmetry. The frieze in Figure 2(b) is a decoration used by the Senufo, people from the northwest of the Ivory Coast. It also is an example of translational symmetry.

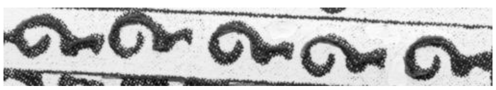


Figure 2(a)

Figure 2(b)

Rotational symmetry is evident in the one-dimensional design inscribed on a wooden drinking cup from Kuba in Zaire in figure 2(c) [8] and the embroidered cloth from the same region in figure 2(d) [9].

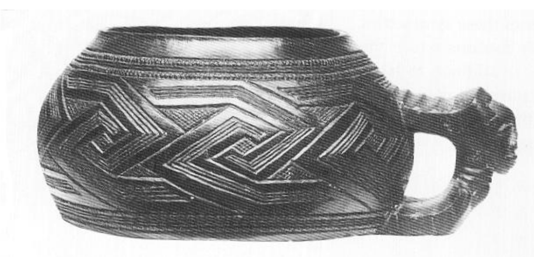


Figure 2(c)

Figure 2(d)

The Mayaka from Angola use the design shown in figure 2(e) to decorate their ceramic ware [3]. The design has the property of vertical reflection. The design in figure 2(f), also done by the Mayaka, has horizontal symmetry [3].

Figure 2(e)

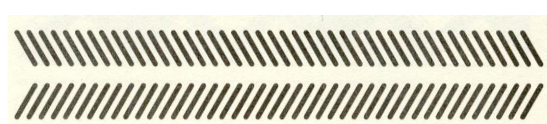


Figure 2(f)

The design on the Kente cloth, woven by the Ashanti and Ewe in Toga (figure 2(g)), is an example of glide reflection [2]. Figure 2(h) is another example of glide reflection in a design used by the Lwimbi-Ngangela from Angola [3].

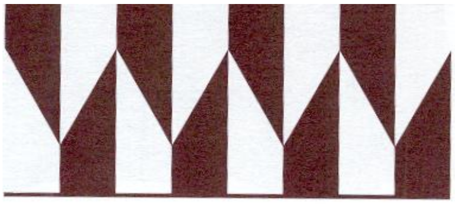


Figure 2(g)

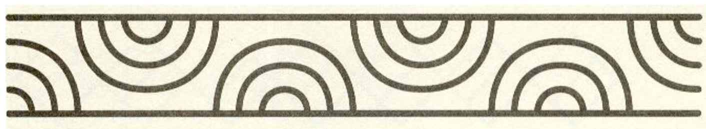


Figure 2(h)

A sample of cloth from Kuba, Zaire (figure 2(i)), displays horizontal and vertical reflection [8]. A piece of fabric from Kuba in Zaire (figure 2(j)) shows glide reflection and half-turn rotation [10].

Figure 2(i)

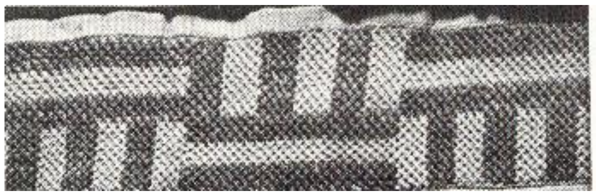


Figure 2(j)

3. Symmetry in Finite Designs

Finite designs are those that admit no translation or glide reflection. A finite design can therefore have only reflectional and/or rotational symmetry. Such designs fall into two classes. The first group includes those patterns that are "cyclic". Cyclic designs have rotational symmetry of order n, where n is some integer, but these designs have no mirror symmetry [8]. Patterns with dihedral symmetry have reflectional symmetry as well as rotational symmetry. Figure 3(a) shows a pattern from Oshogbo in Nigeria. In this design, one can identify rotational symmetry and reflectional symmetry [2].

Both rotational and fourfold reflectional symmetry are evident in figure 3(b), which shows a typical plaster design from the homes of the Swahili in Kenya [2].

Figure 3(a)

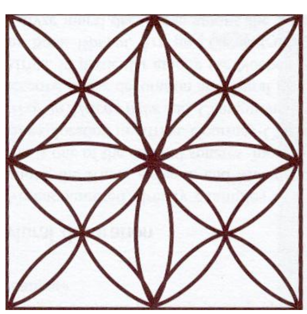


Figure 3(b)

An interesting exercise for students would be to attempt to construct this design using a compass and straight edge only. A possible construction might proceed as follows

Draw a grid of squares as shown. The point C represents the center of the design. The construction proceeds by drawing an arc through three points as shown in figure 3(c). (Students will find that Geometers Sketchpad is particularly useful for creating this design.)

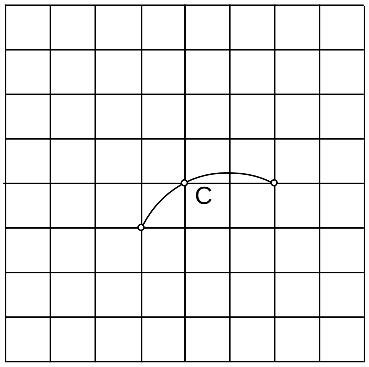


Figure 3(c)

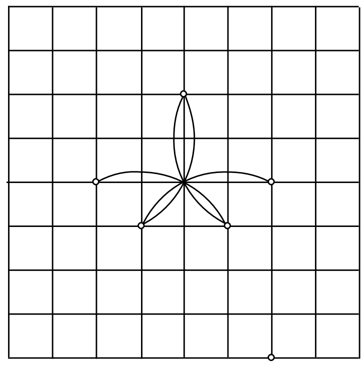


Figure 3(d)

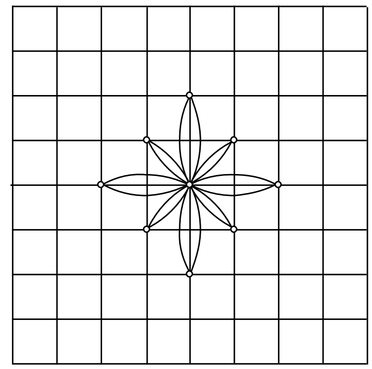


Figure 3(e)

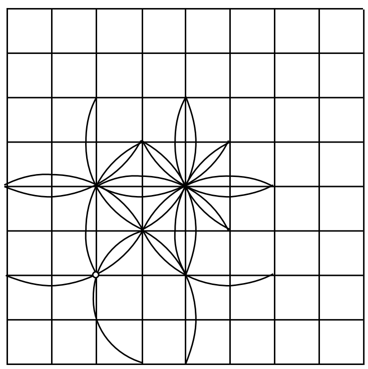


Figure 3(f)

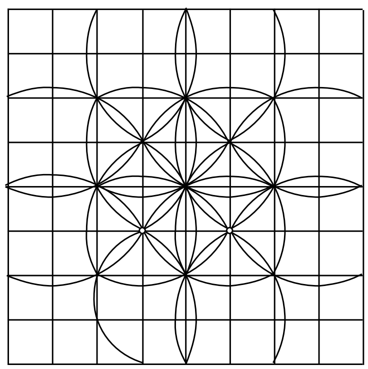


Figure 3(g)

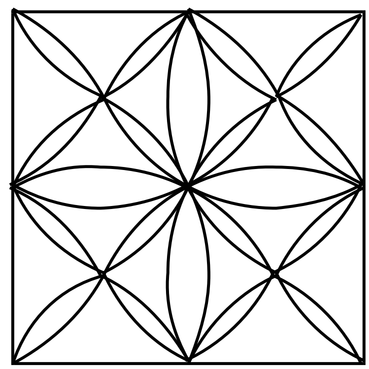


Figure 3(h)

Figure 3(i) and (j) show wood carving designs, creations of the Kuba from central Zaire. Each is an example of rotational symmetry [2].

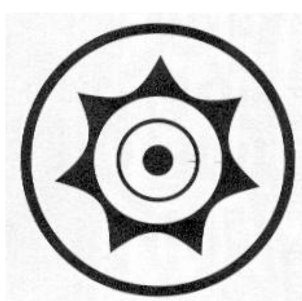


Figure 3(i)

Figure 3(j)

Figure 3(i) has dihedral symmetry of order 14 and figure 3(j) has cyclic symmetry of order 11. Figures which are based on regular heptagon and 11-gon are rare, as are others based on dividing a circle in ways that cannot be constructed using a straightedge and compass in the classical ways. Perhaps, the reason for not seeing these designs in other cultures in Africa, especially the Middle East, is that these constructions are impossible using only a compass and a straight edge [6]. The construction is therefore based on estimation, and most likely relates to the sacred properties of numbers rather than the use of a compass for creating interesting designs.

4. Geometric Constructions Using a Compass and Straight Edge Only

Almost all early constructions related to mathematics in North Africa and the Middle East, was done using a compass and straight edge only. This was a challenge inherited from the Greeks. It enabled them to construct geometric figures using only these two simple tools. Today with the advent of computers, students do not experience the fun or the challenge of doing these constructions. The following examples of African art allow an opportunity to introduce students to some of these techniques. At this point, it may be interesting to point out to students that, while ancient mathematicians discovered how to construct certain regular polygons such as the pentagon and decagon using compass and straight edge only, they were not successful in constructing the heptagon and nonagon using this method. Karl Friedrich Gauss proved that the construction of certain regular polygons is impossible using this method. He proved that the construction of a regular polygon having an odd number of sides is possible only when that number is either a *Fermat number*, a prime number of the form $2^k + 1$, where $k = 2^n$, or is made up by multiplying together different Fermat primes. Such a construction is not possible for a heptagon or nonagon.

The Tuareg women in Niger decorate the camel saddlebags with leather appliqué designs. Amongst these is the design shown in figure 4(a) [2].

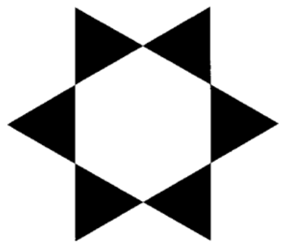


Figure 4(a)

Students can construct this design as follows. Draw a circle and construct a hexagon using the radius of the circle as the length of the side of the hexagon. On each side of the hexagon, construct an equilateral triangle. The resulting construction gives the design shown.

Figure 4(b) is a design found on a wooden mask of the Ligbe in Cote d'Ivoire [2]. Figure 4(c) is a design found on a wooden pilaster used by the Dogon in Mali [2]. Both are pentagonal figures and can be used to introduce the construction of a regular pentagon.

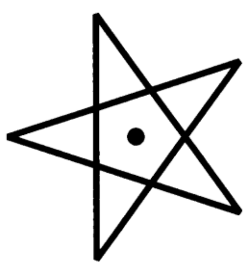


Figure 4(b)

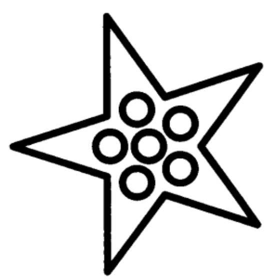


Figure 4(c)

The division of a circle into five equal segments can be accomplished using a ruler and compass only. The method described here uses the Golden Cut on the radius of a circle. The Golden Cut on a line segment OA is the point G on OA such that $\frac{OA}{OG} = \frac{OG}{GA}$. Using this Golden Cut, it is possible to divide the circumference into ten equal sections. Joining every second point leads to the pentagon.

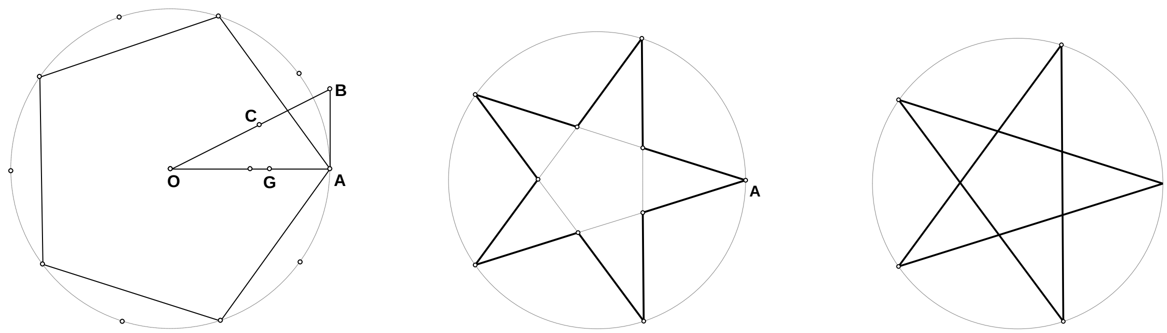


Figure 4(d)

Figure 4(d) showing the construction of a pentagon using the Golden Cut, the construction for design 4(b), and the construction of the pentagram in 4(c). The midpoint of the radius OA is found. At A AB is drawn perpendicular to OA and equal in length to half the length of the radius. B and O are joined and point C is found on OB such that BC = BA. From point O, an arc of radius OC is drawn to cut OA in G. the length OG divides the circle into 10 equal segments.

5. Proofs of Geometry Theorems Using African Artwork

In his book *Geometry from Africa Mathematical and Educational Explorations*, Paul Gerdes points out how designs that appear in different cultures in Africa can be used to prove geometry theorems. He uses a simple headrest decoration from Mozambique to prove the Theorem of Pythagoras as follows.

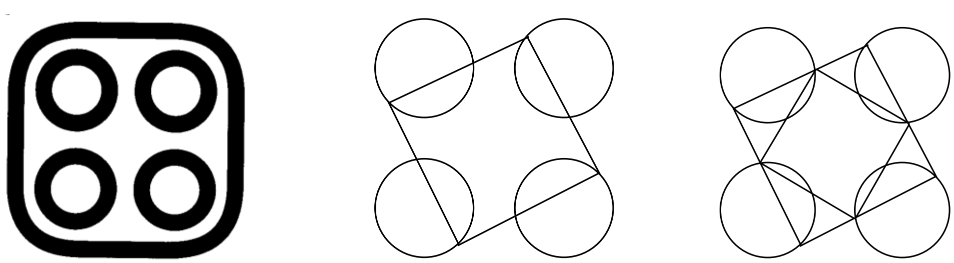
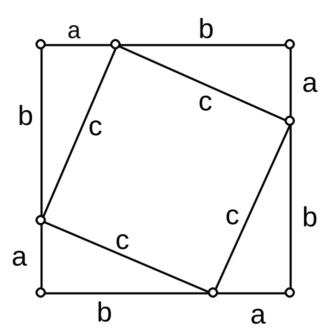


Figure 5(a)

A point is located on one circle. Three points are located on each of the next three circles by a quarter-turn rotation about the center of the four-circle configuration. The four points on the circles are linked by straight-line segments to obtain a square. The points of intersection of these segments with the circle are also joined by segments to form another inner square. Let a, b, and c denote the sides of the triangle.



Let A = area of the large square. Then $A = (a + b)^2$

$$= a^2 + 2ab + b^2 . \quad \text{(i)}$$

However, the area of the square is also equal to the sum of the area of the area of the inner square plus four triangles. Thus

$$A = c^2 + 4\,(\tfrac{1}{2}\,ab) = c^2 + 2ab. \quad \text{(ii)}$$

Combining (i) and (ii) we get $\quad a^2 + b^2 = c^2$.

The decoration in figure 5(b) is on a calabash from the Hona tribe in Nigeria [2]. It can be used to introduce students to the construction of a (8,3) star (figure 5(c)) and then shows the underlying star of the decoration (figure 5(d)). It also can be used to introduce the theorem of Thales. Thales is credited with five theorems of elementary geometry. These are:

(i) *A circle is bisected by any diameter.*
(ii) *The base angles of an isosceles triangle are equal.*
(iii) *The angles between two intersecting straight lines are equal.*
(iv) *Two triangles are congruent if they have two angles and one side congruent.*
(v) *An angle in a semicircle is a right angle.*

It is the fifth theorem that is commonly known as *Thales' Theorem*.

Figure 5(b)

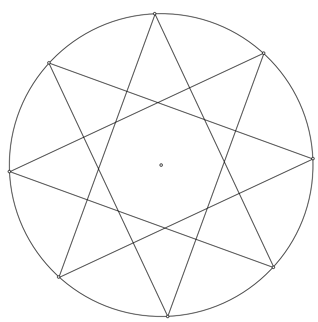


Figure 5(c)

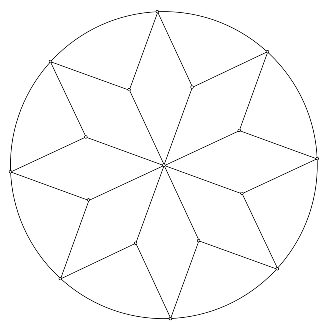


Figure 5(d)

As an introductory activity, students can be asked to measure the size of the angle $\angle ACB$ supported by the diameter on the circumference of the semicircle created by this decoration as shown in figure 5(e). Then several arbitrary points such as *D* can be provided on the circle to observe that Thales' Theorem holds for any such angle in a semicircle.

Finally, a formal proof of the theorem can be given as follows:

$OA = OB = OC$ ....Radii of a circle, which implies that

$\angle OBC = \angle OCB = \alpha$ and $\angle OAC = \angle OCA = \beta$

Then $\angle COA = 180^o - 2\beta$ and $\angle COB = 180^o - 2\alpha$

Since $\angle$COA and $\angle COB$ are supplementary angles,

$(180^o - 2\alpha) + (180^o - 2\beta) = 180^o$

Hence, $\alpha + \beta = 90^o$, which implies that $\angle ACB = 90^o$

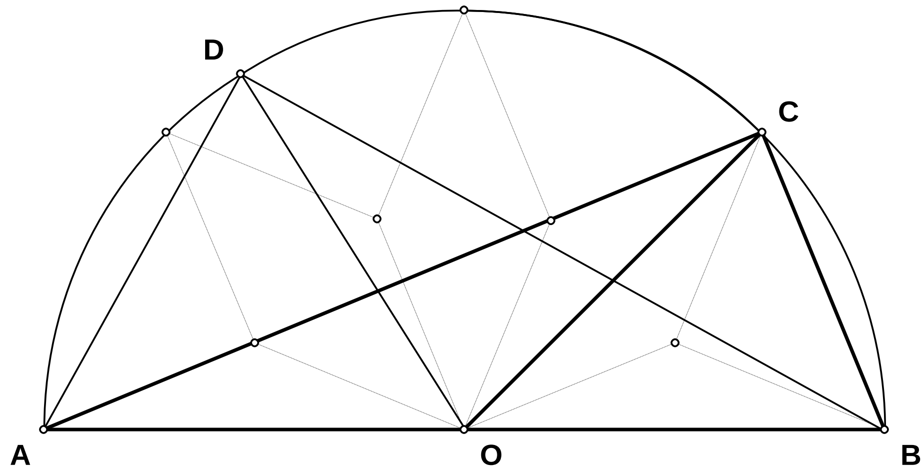


Figure 5(e)

6. Conclusion

In its Principles and Standards for School Mathematics, the National Council for Teachers of Mathematics suggests that geometry offers a means of describing, analyzing, and understanding the world and seeing beauty in its structures [4]. The combination of African art and geometry offers new possibilities for introducing certain geometric concepts while at the same time presenting an opportunity for students to become acquainted with cultures other than their own.

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