Bridges
Mathematical Connections
in Art, Music, and Science

Finding Fibonacci: An Interdisciplinary Liberal Arts Course Based on Mathematical Patterns

Margie Ribble
Mathematics and Computer Science
Maryville College
Maryville, TN 37804
e-mail: ribble@maryvillecollege.edu

Abstract

Integrating mathematics into other disciplines is one of the challenges of mathematics education at all levels. In an undergraduate liberal arts setting, it is particularly desirable to demonstrate connections between diverse areas of the curriculum such as art, music, science, and mathematics. This paper will describe a course that I designed and taught at Maryville College, a four-year private liberal arts college located in Maryville, Tennessee. The course, "Finding Fibonacci," seeks to demonstrate the interrelatedness of a variety of subject areas through mathematical patterns, particularly the Fibonacci sequence.

1. Rationale for the Course

Historically, mathematics has been central to the liberal arts: the *quadrivium* consisted of arithmetic, geometry, astronomy, and music. Together with the *trivium* (logic, grammar, and rhetoric), these subjects made up the classical liberal arts curriculum. Connections between mathematics and philosophy are well-known, and mathematics obviously provides the basis for many scientific discoveries. Therefore it is persuasive for an interdisciplinary mathematics course to have an important place in the curriculum of today's liberal arts college.

The Fibonacci sequence has long been a source of great interest for mathematicians, both because of its number theoretic properties and because of its connections with art, architecture, music, and botany. Its origin in Fibonacci's writing provides an avenue to the history of the Middle Ages, both culturally and scientifically. There are even theories about Fibonacci patterns in the stock market; limericks provide a link to poetry and literature.

Maryville College, in its 1996 revision of the general education curriculum, provided for a senior capstone course known as a Senior Seminar. Faculty were invited to propose topics for such a course under the following guidelines:

> This course should provide the student with the skills and the opportunity to integrate across at least two modes of inquiry. The course is interdisciplinary in nature. It should follow a thematic approach that examines topics from across two divisions and includes global perspectives. Assignments should include use of primary sources, such as texts, films, and art works. Also, the course should provide the student with opportunities to refine oral communication skills beyond classroom discussion. While some offerings could be developed by teams, the expectation is that individual faculty will model the integration of modes of inquiry [1].

Students who began their college work in the fall of 1996 were required to choose a Senior Seminar course during the 1999-2000 school year. Topics of these first seminar courses were: "Issues in Native North America," "The Worth of Work: Labor and Human Vocation in Economics and Literature," "Cosmos, Cultures, and Clocks: Explorations of Time on the Eve of the Third Millennium," "Death, Mourning, and Madness," and my course, "Finding Fibonacci." A central component of all the courses is the requirement that students make a substantial oral presentation on a topic related to the course content.

2. The Course

"Finding Fibonacci: An Exploration of Connections Between Mathematics, History, Art, Music, Literature, Nature, and Economics" was first offered in the Spring 2000 semester and again in the Spring 2001 term. The goals of the course, as outlined in the syllabus, were:

(1) Willingness to explore mathematical patterns and to find them in the arts, humanities, natural sciences, and social sciences.
(2) Oral communication skills that enable effective comprehension, analysis, and expression.
(3) The integration of the scientific, artistic, and humanistic modes of inquiry.
(4) Increased interest and fluency in mathematics.

I compiled a textbook containing readings, assignments, mathematics exercises, suggested topics for student presentations, and sources. An outline of the topics included in the course is given in the textbook Table of Contents:

2.12 – Fractal Geometry

Unit 3 – Art and Architecture
- 3.1 – The Pyramids of Egypt
- 3.2 – The Parthenon
- 3.3 – Leonardo da Vinci

Unit 4 – Music
- 4.1 – Applications of Fibonacci Numbers in Music
- 4.2 – Music of Mozart
- 4.3 – Music of Bartok

Unit 5 – Literature
- 5.1 – Poetry
- 5.2 – Limericks

Unit 6 – Nature
- 6.1 – Plant Growth
- 6.2 – Logarithmic Spirals
- 6.3 – The Human Body
- 6.4 – Reproduction of Rabbits and Bees
- 6.5 – Astronomy

Unit 7 – Economics and Management
- 7.1 – Patterns in the Stock Market and Commodities Trading
- 7.2 – Management Science

Unit 8 – Philosophical Reflections
- 8.1 – Why Is the Golden Ratio Appealing?
- 8.2 – What Factors Contribute to the Making of a Genius
- 8.3 – What Does All This Mean?

Though not all of these topics were covered in class sessions, all readings were required. Many topics were addressed in student presentations at the end of the course. The course syllabus, appended to this paper, gives more detail about requirements and schedule of topics.

A wide variety of instructional strategies kept the classes interesting and varied. An introduction to the life and times of Leonardo Fibonacci included a look at medieval Pisa and Leonardo's contributions to mathematics. I created a number of brief illustrated Power Point presentations—one on Fibonacci the man, one on Fibonacci numbers in art and architecture, and one on Fibonacci numbers in nature. Discovery learning took place in counting parallel rows on pine cones, pineapples, and artichokes, and finding sequential Fibonacci numbers. Spiral formations in nature were compared to the spiral constructed using successive Fibonacci numbers of squares. We measured each other to observe the approximate golden ratio in the human body. Two biology majors prepared a special presentation on phyllotaxis (patterns in plant growth) and related it to Fibonacci ratios. After Fibonacci numbers were discovered in the number of lines and accents of limericks, each student was required to write an original limerick. Guest speakers shared their expertise in subjects such as the stock market, Leonardo da Vinci, and the music of Mozart. Videotapes added background to such topics as the Middle Ages, the golden ratio in the phrasing of Bach's music, the Leaning Tower of Pisa (constructed during Fibonacci's lifetime), and the Egyptian pyramids. A favorite among the students was "Donald in Mathmagic Land," a 1940's animated Disney video which gives considerable attention to the golden section.

We generally interspersed mathematics lessons with other topics. The relationship between Fibonacci numbers and the golden ratio (Φ) was discovered in an early lesson. We observed ratios of adjacent Fibonacci numbers approaching this magic number, and then, using closed-form expressions for the nth and $(n$-1)th Fibonacci numbers, showed that the limit of such a ratio as n approaches infinity is actually Φ or $\left(\dfrac{1+\sqrt{5}}{2}\right)$. Using worksheets from Dale Seymour's *Visual Patterns in Pascal's Triangle* [4], we learned about binomial coefficients and discovered numerous patterns including consecutive Fibonacci numbers on the famous arithmetic triangle. Other sequences were investigated including arithmetic, geometric, Lucas, and Tribonacci. Students were particularly interested in discovering patterns in the Fibonacci sequence itself. They were asked to find a formula for the sum of the first n Fibonacci numbers, the sum of the squares of the first n Fibonacci numbers, the sum of the first n Fibonacci numbers with even indices, etc. The "brute force" method was used to generate several numbers and then observe the pattern.

This pattern exploration provided a number of serendipitous moments in the class. I particularly relished the times when a student would observe a new mathematical pattern. For example, this semester I was explaining the method of constructing Pythagorean triples from four consecutive Fibonacci numers, a well-known result:

$$(F_n F_{n+3})^2 + (2F_{n+1}F_{n+2})^2 = \text{a perfect square of an integer } (c^2)$$

After we wrote out several examples, I asked the class to look at the number c and see if they could discover anything remarkable about it, expecting that they would observe that it is another Fibonacci number, and possibly figure out which Fibonacci number (F_{2n+3}). One student, however, immediately observed that the number c was the difference in the products $F_{n+2}F_{n+3}$ and $F_n F_{n+1}$. For example, if the sequence of four Fibonacci numbers 2, 3, 5, and 8 is used, the Pythagorean triples are 16, 30, and 34. He noted that 34 is the difference between 5·8 (40) and 2·3 (6). Another student was able to prove this conjecture algebraically, using *a*, *b*, *a+b*, and *a+2b* as the sequence of any four Fibonacci numbers. We observed as a class that this sequence is actually *any* four numbers in a Fibonacci-like sequence, such as 5, 6, 11, and 17; therefore the pattern holds in a more general way. We declared this the Wheatley-Ballard Theorem after the students responsible for the conjecture and proof.

Although Fibonacci numbers and the golden section can be observed in many areas of life, students were encouraged to be skeptical about some of the findings. For example, the ratio of numbers of measures of the exposition section and development/recapitulation section of Mozart piano sonatas (those movements in the sonata-allegro form) was discovered by John Putz [3] to approximate the golden ratio. Musicologists do not believe this was purposeful on Mozart's part, but was a result of his innate sense of balance. We listened to a recording of Mozart's Symphony No. 40 in G minor (K 550) which is often said to contain the most perfect first movement in music. Students followed the score and counted measures in the two sections. The golden ratio was nowhere to be found!

Student presentations provided insights into related areas not specifically covered in class activities. Each student was to teach the class for ten minutes on a topic of his or her choice related to course material. It was strongly suggested (and the grade depended on it!) that some audio-visual aid and/or class activity be incorporated. Presentations ranged from "The Golden Ratio in Baseball" to "Fibonacci Numbers in Astronomy." Other topics have included Gothic architecture, Edward Lear and limericks, Lewis Carroll, the history of zero, Mondrian's art, Egyptian pyramids, the Mozart effect, and the Parthenon in Nashville, Tennessee.

The course has been well received by students who participated. Surprisingly, very few expressed anxiety about the mathematics topics, although I suppose one would not expect a math-phobic person to elect a course with math content. Typical comments on the standard Maryville College course evaluation forms were:

"A whole new look into life."
"This course made mathematics much more interesting to learn."
"I found math, a subject I usually dread, to be fun and interesting!"
"A terrific class to finish up and sum up your MC education!"
"Good job of integrating all subjects into the topic of Fibonacci."

On a 10-question Likert scale on this same evaluation form, students rated the course from 1 (lowest) to 5 (highest) in ten categories. In the pilot offering of this course, the average rating was 4.6, compared to an average rating for all Maryville College courses of 4.2.

An excerpt from one student's final essay indicates that course goals were achieved: "I was amazed to see how the Fibonacci sequence and the golden ratio showed up time and again in fields other than mathematics....Many of the disciplines offered at Maryville College seem distinct in themselves, but this course has created a link between many of them. The result is the realization that possibly everything has some special bond, and the knowledge that is available is endless."

3. Conclusion

The course described here is one example of an attempt to show connections between mathematics and other areas of the curriculum. These ideas go beyond the "applied mathematics" courses often included in a college's general education curriculum. I would encourage faculty members at liberal arts institutions to explore the possibilities of designing other course offerings that would similarly integrate mathematics into diverse areas of the curriculum. This integration is both valid and necessary in today's world. A quote from *Universal Patterns*, by Rochelle Newman and Martha Boles [2], speaks eloquently to this need:

> In this world of overspecialization, much of education deals with discrete bits of information rather than large systems. People, therefore, are not trained to find connections. Without connections, value systems are difficult to develop. In the evolution of civilization, Art and Mathematics are disciplines that have been seen as polarities without connection. Yet, in fact, they are the left and right hand of cultural advance: one is the realm of metaphor, the other, the realm of logic. Our humanness depends upon a place for the fusion of fact and fancy, emotion and reason. Their union allows the human spirit freedom (p. xiv).

References

[1] Guidelines for Senior Seminar, Maryville College.

[2] R. Newman and M. Boles, *Universal Patterns,* Vol. 1 of *The Golden Relationship: Art, Math, and Nature,* Pythagorean Press, Bradford, Massachusetts, 1992.

[3] J. Putz, The Golden Section and the Piano Sonatas of Mozart, *Mathematics Magazine* 68 (1995), 275-282.

[4] D. Seymour, *Visual Patterns in Pascal's Triangle,* Dale Seymour Publications, 1986.

APPENDIX

SENIOR SEMINAR 480-02
"Finding Fibonacci"
Spring 2001

SYLLABUS

Instructor: Dr. Margie Ribble, 122 Sutton, x8275, ribble@maryvillecollege.edu

Textbook: *Finding Fibonacci,* by M. Ribble, available in MC Bookstore ($15).
A scientific calculator is also required.

Description: Students will explore and connect simple mathematical patterns found in the Fibonacci sequence, golden ratio, and Pascal's triangle; will discover how these patterns are found in other areas; and will investigate the historical context of these mathematical discoveries. Patterns in art, architecture, music, literature, nature, economics, and technology will be specifically targeted. Students will be expected to prepare an oral presentation on a related topic of interest.

Schedule: Tuesday, Thursday, 9:30-10:50 a.m., 231 Sutton

Course goals:

(1) Willingness to explore mathematical patterns and to find them in the arts, humanities, natural sciences, and social sciences;

(2) Oral communication skills that enable effective comprehension, analysis, and expression;

(3) The integration of the scientific, artistic, and humanistic modes of inquiry;

(4) Increased interest and fluency in mathematics.

Honesty policy:

A basic assumption in this course is that learning can best be fostered by following the Maryville College Covenant, which encourages all students "to act with integrity in all interactions . . . to encourage and support . . . fellow students as they aspire to be honest in their academic endeavors."

Attendance policy:

- Three unexcused absences, except when they result from the most extraordinary circumstances, will lower the final grade for the course one letter.
- Four or more unexcused absences will lead to a semester grade of "F."
- In the case of an unexcused absence, no opportunity will be given to make up missed work.
- Excused absences include only the following:
  (1) absences for which the instructor has given permission in advance;
  (2) medical absences with a physician's signed excuse; and
  (3) sports absences only as they appear on the official schedule.
  Excused absences in excess of five may result in failure in the course.

Grading:

Each class member will be a member of a team responsible for a 10 to 15-minute presentation on a topic of interest related to course material. In addition, each person will write a thoughtful paper (typed, double-spaced, minimum 2-pages) responding to the question in 8.3 of the textbook. Grades on these projects will comprise 40% of a student's final grade.

Mathematics homework, brief quizzes over reading assignments, and other daily writings and presentations will be graded; these will comprise 20% of a student's grade. The student will keep these papers in a portfolio along with other class materials; the completed portfolio will make up another 20% of a student's final grade.

The remaining 20% of a student's grade will be based on attendance and participation.

A final average in the 90's will be at least A-, in the 80's at least B-, in the 70's at least C-, and in the 60's at least D-.

Tentative Schedule:

2-1		Overview of course, pattern activity
2-6	1.1	Life in the Middle Ages
2-8	1.2	Who was Fibonacci?
2-13	1.2	Mathematics History up to the Middle Ages
2-15	2.1, 2.2	Notation, Sequences and Series
2-20	2.8, 8.1	Golden Ratio
2-22	3.1	Art and Architecture
2-27	3.2, 3.3	Art and Architecture
3-1	2.6	Patterns in the Fibonacci Sequence
3-6	4.1	Music
3-8	4.2	The Music of Mozart
3-13	2.7	Geometric Constructions – Golden Rectangle, Fibonacci Spiral
3-15	2.4	Patterns in Pascal's Triangle
SPRING BREAK		
3-27		NO CLASS – Math Contest
3-29	2.9	Pythagoras and Fibonacci
4-3	5.1, 5.2	Literature – Write a limerick!
4-5	6.1, 6.2	Nature
4-10	6.3-6.5	Nature
4-12	7.1	The Stock Market
4-17		The Leaning Tower of Pisa
4-19 to 5-8		Student Presentations
5-10		Wrap-up, Papers due

Suggested supplemental reading:

Burton, D. (1991). *The history of mathematics: An introduction* (2nd Ed.). Dubuque, IA: William C. Brown Publishers.

Cook, T. (1979). *The curves of life.* New York: Dover Publications, Inc.

Garland, T. (1987). *Fascinating Fibonaccis: Mystery and magic in numbers.* Palo Alto, CA: Dale Seymour Publications.

Ghyka, M. (1977). *The geometry of art and life.* New York: Dover Publications, Inc.

Gies, J. & Gies, F. (1969). *Leonard of Pisa and the new mathematics of the Middle Ages.* New York: Thomas Y. Crowell Company.

Huntley, H. (1970). *The divine proportion.* New York: Dover Publications, Inc.

Knott, Ron. *Fibonacci numbers and the Golden Section* [on-line]. Available: http://www.mcs.surrey.ac.uk/Personal/Rknott/Fibonacci/fib.html.

Newman, R. & Boles, M. (1992). *Universal patterns.* Vol. 1 of *The golden relationship: Art, math, and nature.* Bradford, MA: Pythagorean Press.

Stevens, P. (1974). *Patterns in nature.* Boston: Little, Brown and Company.

Vorob'ev. N. (1961). *Fibonacci numbers* (H. Moss, Trans.). Pergamon Press, Ltd.