**BRIDGES**
**Mathematical Connections**
**in Art, Music, and Science**

# BETTER BY THE DOZEN

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**WHY TEN?** 'Tis a pity that the human body came into being with five fingers on each hand instead of six. Scholars generally agree that early humans developed our numbering system consisting of ten digits by counting on their fingers. Most people have learned to use a system to the base ten (decimals) from an early age and have the impression that this base is somehow fundamental. Actually, choice of the base ten was purely arbitrary.

Any value can be the base of a numbering system, and earlier cultures have used other bases. The binary system (base two) figures almost universally in the internal operations of digital computers. However, numbers to base two are extremely cumbersome to use manually.

**THE DOZEN.** Now if the human hand had six fingers, our numbering system almost certainly would contain twelve (one dozen) digits; and a dozen digits gives us a numbering system that has all the merits of decimals and much, much more. Let's call the new system the *dozimal* system. Sometimes to identify which system we are using, we enclose numbers in parentheses and append subscript "D" or "T" to signify base dozen or base ten respectively. For example:

**$(10)_D = (12)_T$ Meaning one zero to base dozen equals onetwo to base ten.**

We need two new digital characters to implement the new base -- one each to replace ten and eleven. To make going from one base to the other easier, let's spell ten backward and use the last three letters of eleven and designate them by "N" and "V" respectively. Thus in base dozen:

**N (net) = $(10)_T$ and V (ven) = $(11)_T$**

**Comparing the two systems:**

| **Base Dozen** | | **Base Ten** | |
|---|---|---|---|
| one | 1 | one | 1 |
| two | 2 | two | 2 |
| three | 3 | three | 3 |
| four | 4 | four | 4 |
| five | 5 | five | 5 |
| six | 6 | six | 6 |
| seven | 7 | seven | 7 |
| eight | 8 | eight | 8 |
| nine | 9 | nine | 9 |
| net | N | ten | 10 |
| ven | V | eleven | 11 |
| dozen | 10 | twelve | 12 |

Continuing in base dozen (base ten numbers in parentheses): dozenone, 11 (13); dozentwo, 12 (14); dozenthree, 13 (15); dozenfour, 14 (16); dozenfive, 15 (17); dozensix, 16 (18); dozenseven, 17 (19); dozeneight, 18 (20); dozennine, 19 (21); dozenven, 1V (23); two dozen, 20 (24), two dozenone, 21 (25);

two dozentwo, 22 (26); two dozennet, 2N (34); and so on until reaching a dozen dozen or one gross, 100 $(144)_T$. The term "gross" currently means a dozen dozen so why not keep it?

The dozen has long been an accepted unit of quantity, probably for convenience in packaging. One dozen units of approximately equal length and width fit neatly into a more manageable package than do ten units -- Three by four as opposed to two by five. But packaging is the least of reasons for the dozen base.

Now let's write the numbers and the multiplication tables from one through one dozen squared:

**Numbers -- one to one gross**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | N | V | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1N | 1V | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 2N | 2V | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 3N | 3V | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 4N | 4V | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 5N | 5V | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 6N | 6V | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 7N | 7V | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 8N | 8V | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 9N | 9V | N0 |
| N1 | N2 | N3 | N4 | N5 | N6 | N7 | N8 | N9 | NN | NV | V0 |
| V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 | V9 | VN | VV | 100 |

**Multiplication Tables**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | N | V | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | N | V | 10 |
| 2 | 2 | 4 | 6 | 8 | N | 10 | 12 | 14 | 16 | 18 | 1N | 20 |
| 3 | 3 | 6 | 9 | 10 | 13 | 16 | 19 | 20 | 23 | 26 | 29 | 30 |
| 4 | 4 | 8 | 10 | 14 | 18 | 20 | 24 | 28 | 30 | 34 | 38 | 40 |
| 5 | 5 | N | 13 | 18 | 21 | 26 | 2V | 34 | 39 | 42 | 47 | 50 |
| 6 | 6 | 10 | 16 | 20 | 26 | 30 | 36 | 40 | 46 | 50 | 56 | 60 |
| 7 | 7 | 12 | 19 | 24 | 2V | 36 | 41 | 48 | 53 | 5N | 65 | 70 |
| 8 | 8 | 14 | 20 | 28 | 34 | 40 | 48 | 54 | 60 | 68 | 74 | 80 |
| 9 | 9 | 16 | 23 | 30 | 39 | 46 | 53 | 60 | 69 | 76 | 83 | 90 |
| N | N | 18 | 26 | 34 | 42 | 50 | 5N | 68 | 76 | 84 | 92 | N0 |
| V | V | 1N | 29 | 38 | 47 | 56 | 65 | 74 | 83 | 92 | N1 | V0 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | N0 | V0 | 100 |

**FROM DECIMALS TO DOZIMALS.** To convert a number less than one gross from decimal to dozimal, divide the decimal number by 12. The whole number is the first dozimal digit and the remainder is appended. Thus $(45)_T$ divided by 12 is 3 dozen with 9 remaining and equals three dozen nine $(39)_D$. Examples:

| Base Ten | | Base Dozen |
|---|---|---|
| **27 = 12)27 = 2 and 3 remainder** | = | **23 (two dozenthree)** |
| **70 = 12)70 = 5 and 10 remainder** | = | **5N (five dozennet)** |
| **111 = 12)111 = 9 and 3 remainder** | = | **93 (nine dozenthree)** |
| **127 = 12)127 = N and 7 remainder** | = | **N7 (net dozenseven)** |

Converting from dozimal to decimal is just as simple. Jot down the last digit; then multiply the next to last by 12 and add. Thus $(23)_D$ becomes 3 + (2 x 12), or 27. Examples:

| Base Ten | | | | Base Dozen |
|---|---|---|---|---|
| 33 | = | 3x12 + 3 | = | 39 |
| 97 | = | 9x12 + 7 | = | 115 |
| 7V* | = | 7x12 + 11 | = | 95 |
| NN* | = | 10x12 + 10 | = | 130 |

***You must use "10" and "11" for "N" and "V" respectively.**

Converting larger numbers is an extension of the same processes. Remember that a digit in a number is a multiplier, and that the position of that digit in the number determines the power to which you carry the base number. For example the number, 2345 (base ten), can be broken down thus:

| | | |
|---|---|---|
| x $10^3$ | = 2 x 1000 | = 2000 |
| 3 x $10^2$ | = 3 x 100 | = 300 |
| 4 x $10^1$ | = 4 x 10 | = 40 |
| 5 x $10^0$* | = 5 x 1 | = 5 |
| Total | | = 2345 |

***Any number to the zero power equals one**

Now to convert the number to dozimals, let's divide the same number in steps by 12. Each division leaves a whole number with a remainder not exceeding $(11)_T$ or $(V)_D$; the final step shows a quotient of zero with only a remainder:

| | | | |
|---|---|---|---|
| 12)2345 | = 195 | and a remainder of | 5 |
| 12) 195 | = 16 | and a remainder of | 3 |
| 12) 16 | = 1 | and a remainder of | 4 |
| 12)1 | = 0 | and a remainder of | 1 |

The remainders in reverse order, 1435, are the number to base dozen. Any doubts? If so, convert the dozimal number 1435 back to base ten:

| | | |
|---|---|---|
| 1 x $12^3$ | = 1 x 1728 | = 1728 |
| 4 x $12^2$ | = 4 x 144 | = 576 |
| 3 x $12^1$ | = 3 x 12 | = 36 |
| 5 x $12^0$ | = 5 x 1 | = 5 |
| TOTAL | | 2345 |

Now let's convert a dozimal number, 2345, to base ten (by now you may omit spelling out the number 12 to the zero power):

| | | |
|---|---|---|
| 2 x $12^3$ | = 2 x 1728 | = 3456 |
| 3 x $12^2$ | = 3 x 144 | = 432 |
| 4 x $12^1$ | = 4 x 12 | = 48 |
| 5 x | 1 | = 5 |
| TOTAL = | | 3941 |

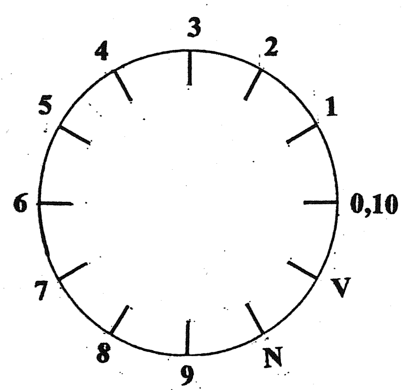
**FRACTIONS.** Because a dozen is evenly divisible by more numbers than is ten, more commonly used fractions convert to rational dozimals than in decimals.

| Fraction | Dozimal | Decimal | Fraction | Dozimal | Decimal | Fraction | Dozimal | Decimal |
|---|---|---|---|---|---|---|---|---|
| 1/2 | = .6 | .5 | 1/6 = | .2 | .16666... | 1/N or 1/10 = | .12497 | .1 |
| 1/3 | = .4.3333... | .3333... | 1/7 = | .186N4... | .142867.. | 1/V or 1/11 = | .1111... | .0909... |
| 1/4 | = .3 | .25 | 1/8 = | .16 | .125 | 1/10 OR 1/12 = | .1 | .08333... |

**1/5 = .24971... .2 1/9 = .14 .1111...**

Obviously, dozimal fractions have fewer irrational numbers than are found with decimals. However, this advantage is hardly pressing.

**THE DOZIMAL CIRCLE.** The real payoff begins with circular measurement. The most commonly used angles in our daily lives are 30°, 45°, 60°, 90°, and multiples of these angles. These angles are also basic be-cause of their functions and relationships in mathematics. Since the circle contains four 90° angles and a 90° angle contains three 30° angles, the numbering base must contain factors 3 and 4 -- in other words one dozen -- in order for the system to work neatly in circular measurement.




**Dozimal Circular Measurement**

Now if you set the dozimal degree (new degree) equal to 30 current degrees, you have a dozen new degrees (ndegs) in a circle. Isn't that neat? Current 30°, 60°, 90° each is a single digit number in dozimals, 1, 2, and 3 ndegs respectively. 45° is 1 and 1/2 or 1.6 ndegs, a perfectly satisfactory number with which to work. The circle at left is divided into a dozen ndegs.

In keeping with the dozimal system, let's make one dozen new minutes (10 nmin) equal 1 ndeg and 1 gross new seconds (100 nsec) equal 1 nmin as in the following table (remember that these numbers are to base dozen.

**100 nsecs = 1 nmin**
**10 nmins = 1 ndeg = 1000 nusec**
**10 ndegs = 1 circle = 100 numin = 10000 nusec**
**Here's how the units compare (degs, min, and sec to base ten):**

**1 ndeg = 30 ; 1 nmin = 12 1/2 min; 1 nsec = 5 5/24 sec**

To convert between ndeg, nmin and nsec, all we need do is move the dozimal point. Take an angle of 6 ndeg, 53 nmin, 17 nsec and convert to dozimals. The angle is 6.5317 ndeg, 653.17 nmin, or 65317 nsec. Try that with the current system; it can't be done.

**LATITUDE, LONGITUDE and TIME.** Let's convert latitude and latitude to dozimal angles. Latitude at the Equator is zero increasing to 3 ndeg North and 3 ndegs South at the North Pole and South Pole respectively. A better way to lay out longitude is to forget about Eastern and Western Hemispheres. Start with the Greenwich Meridian as zero ndeg and proceed eastward ending with a dozen ndegs as we get around again to the Greenwich Meridian. Then convert the day to a dozen new hours (nhrs), a nhr of one gross new minutes (nmin), and a nmin of one gross new seconds (nsecs)? The nhr is twice as long as the current hour. Labor unions, always pressing for a shorter hours, would love the 4-hour day and 20-hour workweek. However, they might be less enthusiastic about rolling out of bed at 3 o'clock in the morning.

Now that we have latitude, longitude, and time in compatible dozimal units, we've eliminated the problems in converting between time zones; local time is Greenwich time plus longitude. A nsec in longitude is a nsec in time; a nhr is the same in both, etc. Think of the saving in time and the improved accuracy for navigators, surveyors, and astronomers working with the simpler system.

**LINEAR MEASURES.** In our present units of measure, one minute of longitude at the equator is one nautical mile (nmi). In their daily operations, scientists, sailors, and aviators use this unit along with nautical miles per hour (knots); but the nautical mile finds little use elsewhere. Current units of measure are a hodgepodge that has very few logical relationships.

For a basic unit of length in dozimals, let's take one dozen nsecs of longitude at the Equator and call it a new mile or numile -- similar to the current nautical mile. Here's how the two units compare using distance along the earth's circumference:

**New: $(10)_D$ ndeg x $(10)_D$ nmin x $(10)_D^2$ nsec = $10^4$ = 1000 new miles or $(20736)_T$ new miles**
**Old: 360 deg x 60 min = $(21600)_T$ nautical miles.**

The ratio of the new to the old is $(20736/21600)_T$ or 24/25. In other words, 24 new miles (numi) equals 25 nautical miles. One nautical nmile is $(6080.27)_T$ ft, so one numi is 25/24 times that value or $(6333.61)_T$ ft. Now we establish new units of length as dozimals of the numile and compare them to current units to get some comparison of length (remember that the new units are in dozimals and the old in decimals):

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.0** | **numi** | = | **6333.6 ft** | | = | **1.9310 km** |
| **.1** | **numi** | = | **1 dozomile** | **= 527.80 ft** | = | **160.91 m** |
| **.01** | **numi** | = | **1 docomile** | **= 43.983 ft** | = | **3.410 m** |
| **.001** | **numi** | = | **1 nft*** | **= 3.6653 ft** | = | **1.1175 m** |
| **.0001** | **numi** | = | **1 nin*** | **= 3.6653 in** | = | **9.312 cm** |
| **.00001** | **numi** | = | **1 dozoinch** | **=.30544 in** | = | **.77582 cm** |
| **.00000** | **numi** | = | **1 docoinch** | **= 25453 in** | = | **.06465 cm** |

***new foot, and new inch respectively.**

Reversing the order and showing dozimals only:

| | | | |
|---|---|---|---|
| **10 dcin#** | = | **1** | **dzin#** |
| **10 dzin#** | = | **1** | **nin** |
| **10 nin** | = | **1** | **nft** |
| **10 nft** | = | **1** | **dcmi#** |
| **10 dcmi** | = | **1** | **dzmi#** |
| **10 dzmi** | = | **1** | **numi** |

**# Docoinch, dozoinch, Docomile and dozomile respectively.**

__AREA, VOLUME, LIQUID MEASURE, AND TEMPERATURE.__ Area, volume, and liquid measures fit well into the dozimal system following in principle the metric system. Let's start with temperature. Make a scale with one gross new degrees equal to the difference between the melting and boiling points of water and call the scale the Dozimal or D scale. Thus $(100$ Dozimal degs$)_D$ = $(100$ Celsius degs.$)_T$, and the ratio of degrees D to degrees C is 144/100 or 36/25. The ratio of degrees F to degrees C is 9/5, or 36/20; so the degree D approaches the degree F in magnitude. An accurate comparison (base ten) is:

**144°D = 100°C = 180°F -- the difference in temperature between melting and boiling.**

The dozoinch is approximately 0.3 inches or 0.8 centimeters. Let's use one cubic dzin (cdzin), approximately equal in volume to 1 cc, as a standard. Now set the weight of 1 cdzin of water equal to one new gram (ngm). To continue, let the amount of heat required to raise the temperature of 1 ngm of water 1°D equal one new calorie (ncal). Comparable to the liter and quart would be the cubic nuinch which is equal to $((10 \text{ nuin})^3)_D$. These units parallel the metric system.

Now let's determine in dozimals the speed, "v", of a falling object after one nsec. (This is numerically equal to the acceleration of gravity, "g", which in base ten is 32 ft per sec. per sec.) Expressing time as "t", the formula for velocity of a falling object is v = gt. Then:

**One nsec = 55/24 secs. Therefore, after 1 nsec the object is falling at a rate of v = 32 x 125/24 = 166.67 ft/nsec**

and 166.67 ft = 49.97 nft which says that the acceleration of gravity in dozimals is 49.97 nft/nsec/nsec, and velocity at the end of a fall of one nsec is 49.97 nft/nsec.

<u>USING THE SYSTEM</u>. Now let's look at speed and rotation. You find that expressing circular motion in dozimals is much simpler than in currently used units. For example, 600 (six gross) revolutions per new minute (r/numin) dozimal units, just shift the dozimal point:

**600.0 r/nmin = 6.0 r/nsec = 60000.0 r/nhr = 600000.0 /day = 6000000.0 ndegday = 600000.0 deg/nhr = 6000.0 ndeg/nmin = 60.0 ndeg/nsec**

Now let's arbitrarily pick a speed of 302 nuft/nsec and see what we get by shifting the dozimal point:

| | | |
|---|---|---|
| **302.0 nft/nsec** | **3020.0 dcmi/nmin** | **3020000.0 nft/nhr** |
| **3020.0 nuin/nsec** | **3020.0 dcmi/nmin** | **3020000.0 nft/nhr** |
| **30200.0 dzin/nsec** | **30200.0 nft/nmin** | **302000.0 dcmi/nhr** |
| **302000.0 dcin/nsec** | **302000.0 nin/nmin** | **30200.0 dzmi/nhr** |
| **30.2 dcmi/nsec** | **3020000.0 dzin/nmin** | **3020.0 numi/nhr** |
| **3.02 dzmi/nsec** | **30200000.0 dcin/nmin** | **30200.0 numi/day** |
| **.302 numi/nsec** | **3020000000.0 dcin/nhr** | **302000.0 dzmi/day** |
| **30.2 numi/nmin** | **302000000.0 dzin/nhr** | **3020000.0 dcmi/day** |
| **302.0 dzmi/nmin** | **30200000.0 nin/nhr** | **3020000.0 dcmi/day** |
| **3020000.0 dcmi/day** | **30200000.0 nft/day** | **302000000.0 nin/day** |

<u>SO WHAT?</u> What can we do with it? Converting our numbering base would be chaos. We can't even get the U.S. to convert to the metric system that is superior to the "English" system -- if we can still call it that. After all, the British finally made the switch.

Converting not only would mean updating all mathematical and astronomical tables and textbooks. All numbers, dates, pages, tables, and chapters in every thing on record would also be out of date. The switch would take decades if not centuries; and during that time, everyone would have to be proficient in both systems -- a lot to expect when so many have so much trouble managing one system.

A small segment of our population believe we, as a civilization, are rapidly annihilating our race with costly pollution, wars, drugs, disease, crime and too much government regulation. Some of them are preparing for the mass exodus and seeking "safe refuges" from the catastrophic events to come. If indeed they are correct, hopefully a few will survive. And if so, perhaps we might leave something to "posterity".

In what time we have left, we could modify a computer to use the base dozen. We could then make a few copies of all pertinent tables and other material and try to preserve them. We could also prepare a globe and maps with the new grid system. Those who do survive would have little left of the material world we now have. And this includes our great libraries and collections of art and written material. Perhaps if we were to have them stash the new material in their refuges, they would have a nucleus to begin a new culture with a new system of numbers and measures. The system would be so superior to ours that they could progress more rapidly and more accurately than possible if they preserved the base ten. []