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Mathematical Connections
in Art, Music, and Science

M. C. Escher's Association with Scientists

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Abstract

Mathematicians, crystallographers, engineers, chemists, and physicists were among the first admirers of Escher's graphic art. Escher felt closer to people in the physical sciences than he did to his fellow artists because of the praise he received from them. Some of Escher's artwork was done more like an engineering project using ruler and compass than in a free spirit mode. Mathematicians continue to promote his work, and they continue to use his periodic patterns of animal figures as clever illustrations of symmetry.

Introduction

For more than 40 years, scientists have been impressed with the graphic art of M. C. Escher, recognizing with fascination the laws of physics contained within his work. Psychologists use his optical illusions and distorted views of life as enchanting examples in the study of vision. Mathematicians continue to use his periodic patterns of animal figures as clever illustrations of translation, rotation and reflection symmetry.

Escher's visual images relate directly to many scientific and mathematical principles. Some of his drawings give visual examples of the infinite process. Many scientists see in his work visual metaphors of their scientific theories. My interest in Escher stems from my many years of teaching geometry and the fact that some of Escher's work gives excellent examples of translation symmetry, rotational symmetry, glide reflection symmetry, and line reflection symmetry. Also, some of his work relates to models in non-Euclidean geometry. For example, the first time I saw a print of Circle Limit III, I said to myself, "that is the most beautiful example that I have ever seen of the Poincare circle model for Hyberbolic Geometry." Little did I realize at the time, that Escher got his original idea for this woodcut from a geometric figure in an article written by the mathematician H. S. M. Coxeter.

A question that might be asked, "Why is this paper being written by a mathematician rather than an artist?" Well for one thing, M. C. Escher is not considered one of the great artists of the world. In 1998, a New York Times art critic stated, "Escher's work tends to be viewed, at best, as art for beginners, an esthetic first love" [1]. Escher's name is usually mentioned in an art history course mainly because of his popularity. Also, people in the physical sciences (mathematicians, chemists, physicists, and crystallographers) were the first admirers of Escher's work, and they continue to enjoy his work [2].

Mathematicians continue to be promoters and admirers of Escher's work. For example, in 1998, I gave a paper at an International Escher Conference in Italy. Members of the Department of Mathematics at the University of Rome 'La Sapienza' organized the conference. Doris Schattschneider, a mathematics professor at Moravian College in Pennsylvania, was also a major organizer of the conference. I consider her to be the world authority on M. C. Escher.

At the above mentioned conference held in Italy, an Art Exhibition was held where contemporary artists displayed their work which they claimed was influenced by Escher. Displayed were examples of impossible objects, variations on tessellations, sculptures of many kinds, reflections in cylinders, reflections in spheres, approaches to infinity, depth perception, periodic space division, multi-dimensional space graphics, fractal tiles, etc. [3]

During the later part of Escher's life, he developed a close association with people in the physical sciences. He gave a lecture at Massachusetts Institute of Technology. The Union of Crystallographers invited him to speak at one of their conferences. Escher often visited his grandchildren in Canada, and during one of these visits, H. S. M. Coxeter of the Mathematics Department at the University of Toronto arranged for Escher to give some lectures on connections between mathematics and art.

The following two quotes demonstrate how M. C. Escher felt about his relationship to mathematicians. Escher said, "...the sad and frustrating fact remains that these days I'm starting to speak a language which is understood by very few people. It makes me feel increasing lonely. After all I no longer belong anywhere. The mathematicians may give me a fatherly pat on the back, but in the end I am only a bungler to them" [4].

Another quote: "I never got a pass mark in math. The funny thing is I seem to latch on to mathematical theories without realizing what is happening. No indeed, I was a pretty poor pupil at school. And just imagine—mathematicians now use my prints to illustrate their books. Fancy me consorting with all these learned folk, unaware of the fact that I'm ignorant about the whole thing" [5].

The Roosevelt Collection

Long before the artwork of M. C. Escher became popular, Cornelius V. S. Roosevelt (1915 – 1991) began collecting Escher's work. He was a grandson of American president Theodore Roosevelt, and a graduate of Massachusetts Institute of Technology in Mining Engineering. For most of his professional career, he worked for the United States Central Intelligence Agency.

Beginning in 1952, Roosevelt began collecting Escher art. The National Gallery of Art in the United States has a collection of over 200 original Escher prints that Cornelius Roosevelt donated to the gallery. To celebrate the centennial of the birth of Escher in 1998, the National Gallery held an Escher art exhibit that drew large crowds, over twice the average for a small National Gallery show. Approximately half of the prints displayed were from the Roosevelt Collection. Roosevelt purchased most of his prints directly from Escher.

Beginning with a written correspondence in 1957, Roosevelt's friendship with Escher developed into a warm and cordial relationship. Roosevelt acted as an advisor for Escher in the United States and took care of many copyright problems for him. People who wanted an Escher print placed in a publication, often went to Roosevelt's residence in Washington, D. C. to look over his collection.

I never met Roosevelt in person, but did talk to him on the telephone several times and corresponded with him. I asked Roosevelt for his picture standing next to some Escher prints, and he sent me a copy of a 1974 edition of *Holland Herald*, a news magazine of the Netherlands, which contained Roosevelt's picture in an article titled "The Roosevelt Eschers."

All the written materials that Roosevelt accumulated were carefully catalogued and placed into binders and these volumes were donated to the National Gallery Library. From a historical point of view, this written material is the most valuable part of the Cornelius Roosevelt Collection. Many of the examples cited in this paper are from this collection.

In a letter dated 24 December 1966, M. C. Escher requested that Roosevelt handle on his behalf inquiries which he received from publishers in the United States who wished to use Escher prints in books and magazines. He repeated his request in a letter dated 16 December 1968. The

correspondence in the Roosevelt Collection contains many letters of requests to have an Escher print placed in a publication. I will cite only a few such requests here. The Roosevelt Collection at the National Gallery of Art Library contains many more examples. The Unitarian Universalist Association requested permission to use the print *Relativity* in a teacher's workbook. They also made some requests to use some prints for a curriculum course for preadolescent children.

In the Roosevelt correspondence there are many letters from well-known publishing houses asking for permission to use an Escher print in a publication. There are also requests from Nobel Laureates and from college professors. Requests were made by Harvard University Biological Labs, the Department of Psychology of Cornell University, a Harvard University psychology professor, American Hotel Association, Colorado State University, etc.

Since so many have given so many different interpretations to Escher's work, perhaps a note might be appropriate as to how he felt about all this. A letter from C. V. S. Roosevelt to John Graham in November 1976 sheds some light on this: "It is always a pleasure to hear from another admirer of the late M. C. Escher, and I will do my best to answer some of your questions. Bear in mind that on several occasions Escher said that he never disagreed with someone who explained what he saw in one of Escher's prints, and many of them were used by Nobel Laureates to explain some point (perhaps in physics, chemistry or mathematics) that Escher never had in mind when he made the print. What he did object to was somebody trying to explain what Escher had in mind when he made the print. In a few cases, he agreed, but in general he thought that those who claimed to recognize Zen Bhuddism or an anticlerical bias in his works were making themselves ridiculous."

Coxeter and the Circle Limit Prints

It is well documented in written correspondence that the mathematician H. S. M. Coxeter influenced Escher's work. The initial ideas for Escher's *Circle Limit* prints came from a drawing in a volume that Coxeter sent him, and Escher recognized this when he wrote on 5 December 1958: "Did I ever thank you for sending me (more than half a year ago) *A Symposium on Symmetry?* Though the text of your article ... is much too learned for a simple plane patterned-man like me, some of your text-illustrations...gave me quite a shock. Since a long time I am interested in patterns with 'motives' getting smaller and smaller till they reach the limit of infinite smallness. The question is relatively simple if the limit is a point in the center of the pattern. Also a line-limit is not new to me, but I was never able to make a pattern in which each 'blot' is getting gradually smaller from a center towards the outside... Are there other systems besides this one to reach a circle-limit? Nevertheless I used your model for a large woodcut (of which I executed only a sector of 120 degrees in wood, which I printed 3x)." Here Escher is talking about *Circle Limit I* for which he carved one-third of the interior of circle from a block of wood (as though someone were having a huge piece of pie) and printed it three times in a circle by rotating the same wood block. This gives the figure rotational symmetry.

Coxeter responded to Escher on 29 December, 1958: "In answer to your question 'Are there other systems besides this one to reach a circle limit? I say yes, infinitely many! This particular pattern is denoted by [4, 6] because there are 4 white and 4 shaded triangles coming together at some points, 6 and 6 at others. But such patterns [p, q] exist for all greater values of p and q, and also for p = 3 and q = 7, 8, 9, ... A different but related pattern, called {p, q}, is obtained by drawing new circles through the 'right-angle' points, where just 2 white and 2 shaded triangles come together. I enclose a spare copy of {3, 7}... After the Colloquium of the Edinburgh Mathematical Society in St. Andrews, my wife and I are hoping to visit Holland at the beginning of August, and we would like very much to come and see you. With all good wishes for 1959, Sincerely yours, H. S. M. Coxeter"

Whenever Coxeter gave Escher a mathematical explanation of anything, Escher always responded that he did not understand. For example, Escher wrote to Martin Gardner on January 30, 1961: "I enclose herewith a copy of my letter to Prof. Coxeter, together with his answer of May 1960. His theoretical explanations are, no doubt, more comprehensible to you than to me. I am and shall ever be a perfect layman in the mathematical field. It is true that I never could have made this picture if I hadn't seen a schematic figure in one of Coxeter's publications, but as soon as he starts to argue abstractly, with formulas, I'm completely lost. I think he won't believe it, but it's a fact."

Impossible Figures and the Penroses

It was Escher who brought drawings of impossible objects to the attention of the general public with his lithographs *Belvedere* (1958), *Ascending and Descending* (1960) and *Waterfall* (1961). The ideas for the prints *Ascending and Descending*, and *Waterfall* were inspired by an article published by L. S. Penrose and R. Penrose (father and son). Escher recognized the Penroses contribution when he wrote to Roosevelt on November 30, 1961: "Waterfall...which is brand new, is also based on an idea of the same two Penroses. It's another of their exciting 'impossible' objects, which I copy here underneath for you."

Roger Penrose, British mathematician and inventor of the now famous Penrose tiles (kites and darts), visited Baarn, and Escher wrote Roosevelt the following about the visit on March 8, 1962: "Young Dr. Roger Penrose, son of the London prof. paid me a very nice visit with his wife. We had so many things to discuss and so much to tell each other that they lost their plane back to England. I am often struck by the simplicity and childish playfulness of most of these leaned scientists and that is why I like them and feel more at my ease with them than with my own colleagues."

Scientific American

Escher's popularity gradually became widespread in the United States because of some national coverage beginning with *Time* magazine in April 1951, *Life* magazine in May 1951, *Saturday Evening Post* in 1961 [6], *Scientific American* magazine in 1961 and in 1966, and *This Week* magazine in 1966. Martin Gardner who wrote a monthly column called "Mathematical Games" in *Scientific American* featured Escher in his April 1966 column. These articles resulted in the artist receiving a large volume of mail concerning his prints. An indication of this is stated in a letter on May 6, 1966 from Escher to Roosevelt: "After Mr. Gardner's article, my customers, especially in America, give me no peace."

Influence of Polya

George Polya, who was a research mathematician, published an article in a crystallographic journal in 1924 that influenced Escher's work. This article pictured examples of the 17 symmetries of the plane sometimes called the wallpaper groups because their resemblance of the periodic patterns in wallpaper. Each figure of Polya's table has translation symmetry but also contains different combinations of line reflection, rotation and glide reflection symmetry to complete the classification. Escher studied the figures, which helped him produce animal shapes that fit into tile-like patterns that cover the plane in a regular interlocking manner [7]. Polya verified his influence on Escher when he wrote the following on August 3, 1977 in a letter to

Doris Schattschneider: "Some time after the publication of my article, I received some drawings from a then unknown Dutch artist accompanied by a nice letter in which he said that my article was very useful to him—the artist was Escher. Unfortunately, in my wartime moving from Switzerland to the U. S. the letter and drawings were lost—your discovery is some consolation for this loss."

Crystallographers

A crystallographer is someone who studies the science of crystal structure and phenomena. Members of the IUC (International Union of Crystallography) were among the early admirers and promoters of Escher's work. P. Terpstra was the first crystallographer to give published recognition to Escher. In 1955 he used an Escher drawing as a front-piece on one of his textbooks. In 1960, Escher gave a lecture to an overflow crowd at the Congress of the IUC in Cambridge, England. Prior to this in 1959, Escher had some visits from crystallographers to his home in Baarn, Holland. Escher wrote the following about these visits: "the nicest thing to have happened to me recently was a visit yesterday afternoon—which is why I couldn't write to you—from a lady, Professor MacGillavry, who lectures on crystallography at the University of Amsterdam. She came with her sister-in-law, who was also somehow interested in divisions of the plane, and the two of them had their eyes glued to my prints from half past two till after half past five. What a pair of smart ladies! It's such a relief to have visitors at last who don't stare at my creatures uncomprehendingly, but who can chuckle with amusement at anything worth chuckling at. How they gazed at some of those prints. A few months ago I received a Belgian-American colleague of hers, a Professor Donnay, who lectures somewhere in the U S, and apparently she put Mrs MacG. on to me. She will try to have me give a lecture in Cambridge in August 1960, where there is a conference of about seven hundred crystallographers. In fact, that Professor Donnay had also mentioned it and it now looks as though it will really happen. There will be an exhibition of my prints as well, and my travelling and hotel expenses will be reimbursed" [8]. Because of the great interest in Escher's work at the Congress, the IUC commissioned Carolina H. MacGillavry to write a book to help crystallography students learn about symmetry using Escher's periodic drawings. Escher was delighted with the idea. Professor MacGillavry worked with Escher and selected a variety of regular periodic plane patterns. She found that some the patterns that crystallographers use in their classification of crystals were missing, and requested Escher to create some new drawings. Escher also improved some of his drawings that were selected by MacGillavry [9]. The book was published in 1965 and reprints of it today are titled *Fantasy & Symmetry: The Periodic Drawings of M. C. Escher.*

Other Scientists

Scientists from many different fields have related their abstract scientific theories to Escher's work. The fact that so many people have used his prints in their books, articles, and lectures attest to Escher's genius in being able to express graphically so many scientific ideas. Escher did not know he was expressing these theories, he just created what was in his head. First, a few examples given by Doris Schattschneider from her book *Visions of Symmetry.*

In 1957, Escher's drawing of the horsemen was placed on the cover of a book by Nobel-prize-winning physicst C. N. Yang.

Physist J. H. de Boer related his theories about the structure of matter to Escher's *Sky and Water.* He said, "The question which of the two, atoms or vacancies, is the most important in crystals, is well-demonstrated by the Dutch artist M. C. Escher."

The biologist E. P. Whitehead related close packing in molecular biology to Escher's metamorphosis drawings.

Nobel-prize-winning chemist Melvin Calvin used Escher's print *Verbum* to illustrate his thinking on chemical evolution. He said that it "struck me as representing, in artistic form, the essence of what I had been thinking about the nature of evolution and living processes...."

J. W. Wagenaar used Escher's prints *Sky and Water*, and *Day and Night* in a 1952 article "The Importance of the Relationship 'Figure and Ground' in Fast Traffic."

In 1968, Jane Abercrombie used Escher's prints *Sky and Water*, *Metamorphosis II*, and *Liberation* for a lecture on group analytic psychotherapy. She said this about the first print, "These designs picture the closely interlocking relationships which are established in a group, and the way the participants draw apart, reestablish their own contours, and become individuals again as they leave it" [10].

My bookshelf contains a popular book "Mathematics A Human Endeavor" by Harold R. Jacobs, which was published by W. H. Freeman and Company in 1970. The front and back covers of this book show Escher's woodcut *Horseman*.

I have always thought that Escher's lithograph *Waterfall* is an excellent example of a machine of perpetual motion. A retired colleague of mine, Albert Read of the Physics Department, thinks that *Waterfall* is an excellent example of "conservation of energy." Professor Read received permission directly from Escher to place a copy of *Waterfall* in one of his publications. A letter dated March 11, 1970 from M. C. Escher to A. J. Read says: "I am giving you permission to print 'Waterfall', however yust this once, in the physics textbook you are writing on condition that you will send me a copy of the book." The spelling of "yust" is left as in the original letter.

A few years ago I was wearing an Escher tie with a butterfly pattern (Figure 1), and a friend of mine, Greg Weed, a PhD chemist who works for DuPont, after studying the tie said, "I see polymers in your tie, does that make sense?" I responded immediately with "yes." Even though I know very little about chemical polymers, and I do not know much about the background history of the pattern in the tie, I still feel confident that my answer was correct, "yes it does make sense for you to see polymer patterns in that tie." The only thing I know about the construction of the butterfly pattern is that it was initially drawn on isometric paper [11]. Isometric paper is graph paper made up of small equilateral triangles.

Mathematicians have used Escher's repeating symmetrical patterns that tessellate (fill the plane without gaps) as examples in formal group theory. They let G be the group of symmetries when color is ignored, and H the normal subgroup of symmetries that preserves all colors. Then it turns out that G/H is a group of order N, where N is the number of different colors or a multiple of the number of colors. The original colored butterfly pattern shown in Figure 1 has 3 different colors, and has quotient group P6/P3 which has order 6. The order or degree of a finite group is the number of elements in the group. The groups P6 and P3 are individually infinite groups whose elements are rotations and translations of butterflies in the figure [12].

Computer Scientists have written programs that will create Escher-like repeating patterns that will tile or tessellate the plane. Douglas Dunham of the University of Minnesota at Duluth has used a computer plotter to produce beautiful hyperbolic patterns like Escher's "Circle Limit" Series [13].

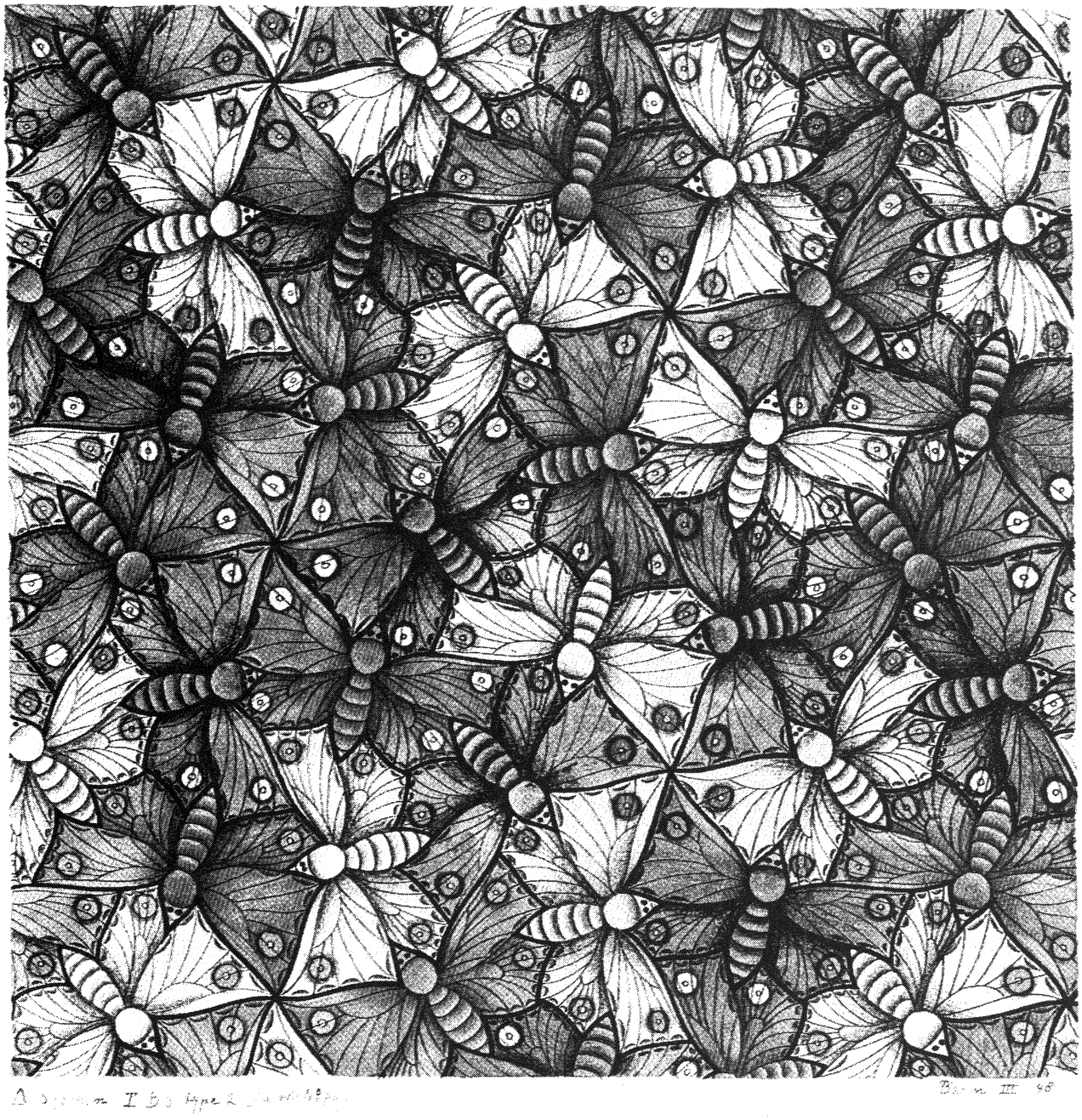

Figure 1: *Escher's Butterfly Pattern. The original is colored in red, blue, and green so that no two adjacent butterflies have the same color. Two colors alternate around the butterfly wing tips to form a six-fold center of rotation (red & blue around one point, red & green around another point, and blue & green around another point). Copyright-fee paid to Cordon Art B. V. of Baarn, Holland to include this print in this article. Cordon Art is the exclusive worldwide representative of the M. C. Escher copyrights.*

References

[1] Roberta Smith, "Just a Nonartist in the Art World, But Endlessly Seen and Cited", *New York Times*, page E1. January 21, 1998.

[2] J. Taylor Hollist, "Escher Correspondence in the Roosevelt Collection", *Leonardo,* Vol. 24, No. 3, pages 329-331. 1991.

[3] Michele Emmer & Doris Schattschneider, *Homage To Escher*, (Department of Mathematics, University of Rome 'La Sapienza') June 1998.

[4] J. L. Locher, *M. C. Escher: His Life and Complete Graphic Works* (New York: Harry N. Abrams), page 93. 1981.

[5] Bruno Ernst, *The Magic Mirror of M. C. Escher* (New York: Barnes & Noble Books), page 24. 1984.

[6] See [4] pages 67, 120.

[7] Doris Schattschneider, *Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M. C. Escher* (New York: W. H. Freeman), pages 23-26. 1990.

[8] See [4] page 94.

[9] See [7] pages 42, 72, 103, 115, 275.

[10] See [7] pages 277-281.

[11] See [7] page 114.

[12] H. S. M. Coxeter, "Coloured Symmetry", *M. C. Escher: Art and Science* (New York: Elsevier Science Publishing Company) pages 15 - 33. 1986.

[13] Douglas J. Dunham, "Creating Hyperbolic Escher Patterns", *M. C. Escher: Art and Science* (New York: Elseveir Science Publishing Company) pages 241- 248. 1986.