BRIDGES
Mathematical Connections
in Art, Music, and Science

The Square, the Circle and the Golden Proportion
A New Class of Geometrical Constructions

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Introduction

The reason behind taking another look at the number Phi is its overwhelming appearance in art, nature and mathematics [1,2,3]. I feel that such a power must have a deep basis. As a result of this investigation I have discovered a new world of geometrical relationships residing within the square and the circle. This picture essay can be read as an example of how complexity emerges inexorably from simplicity.

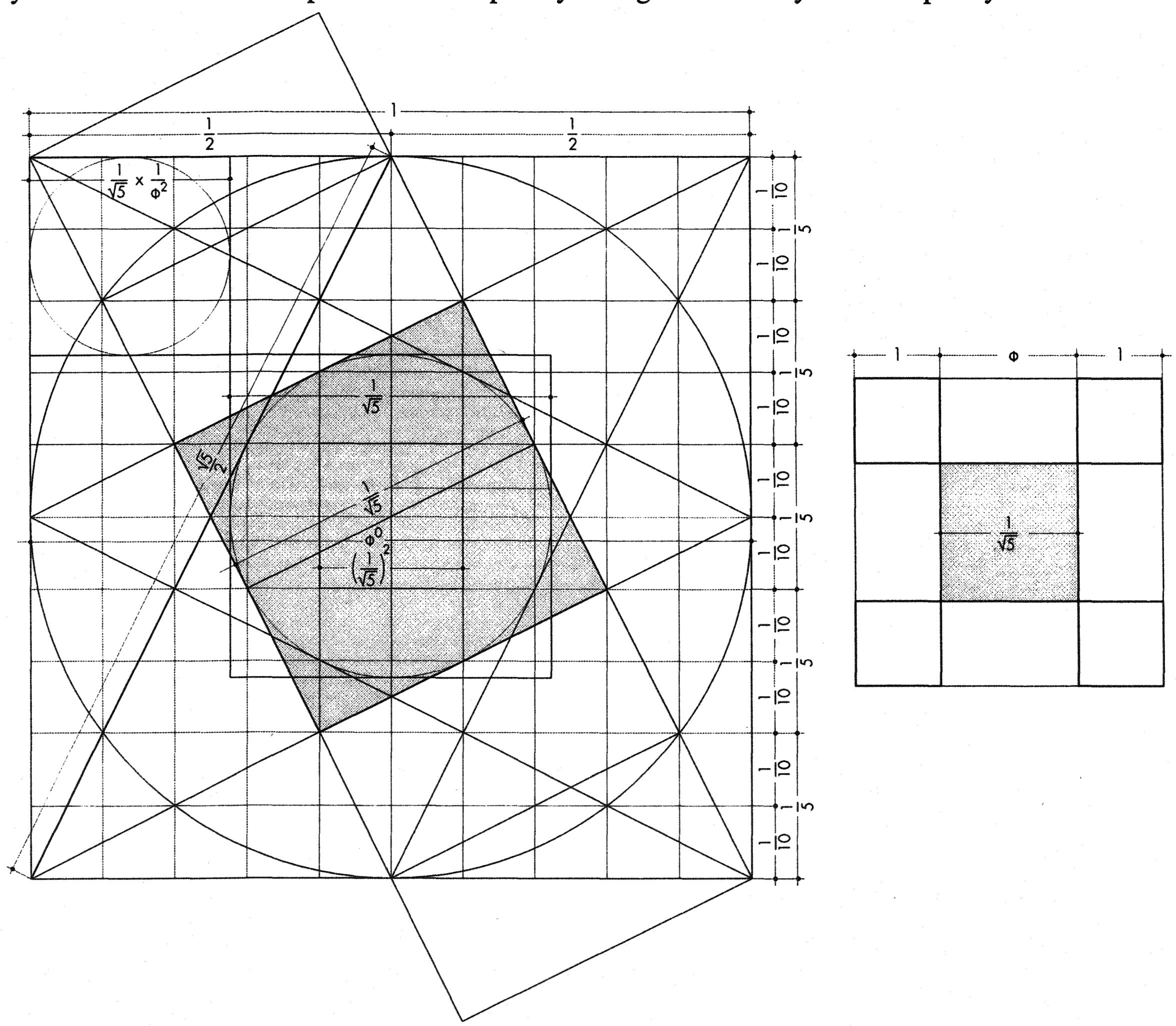

Figure 1a and 1b: *The square, the circle and 8 (√5/2) diagonals forming eight pointed star. Notice how the 10 x 10 grid appears naturally from it. The shaded square in the middle has side (1/√5). When rotated so as to be vertical, the golden proportion appears as shown to the right and above. Many properties of this star have been investigated by T. Brunes and J. Kappraff [4,5].*

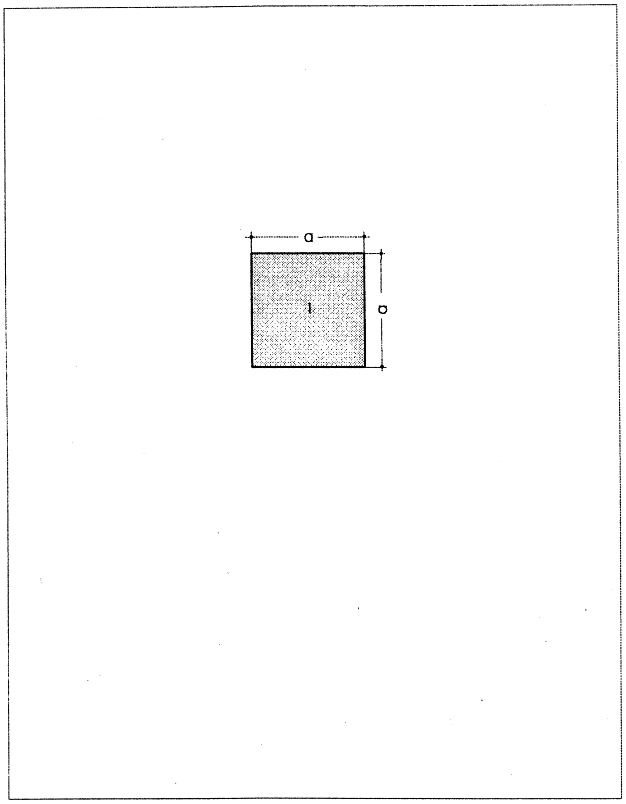

Figure 2: *A square.*

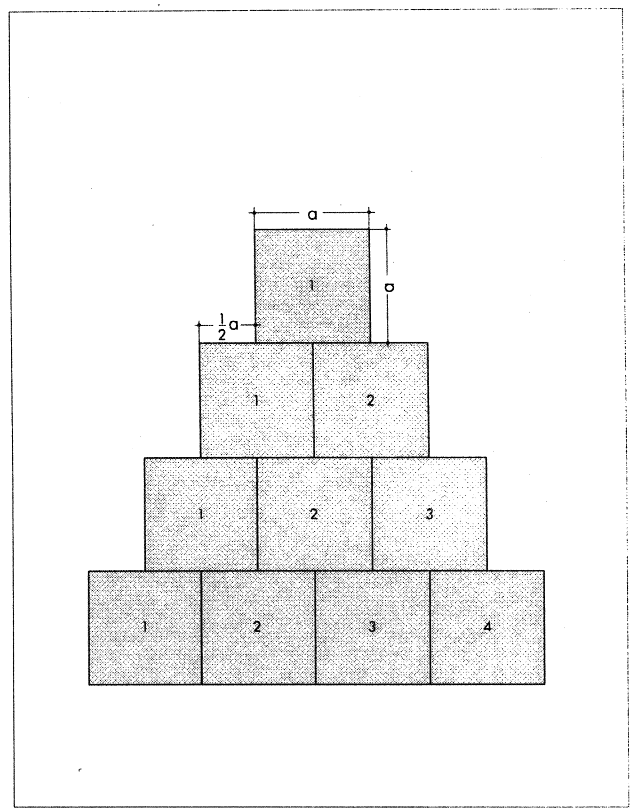

Figure 3: *10 squares.*

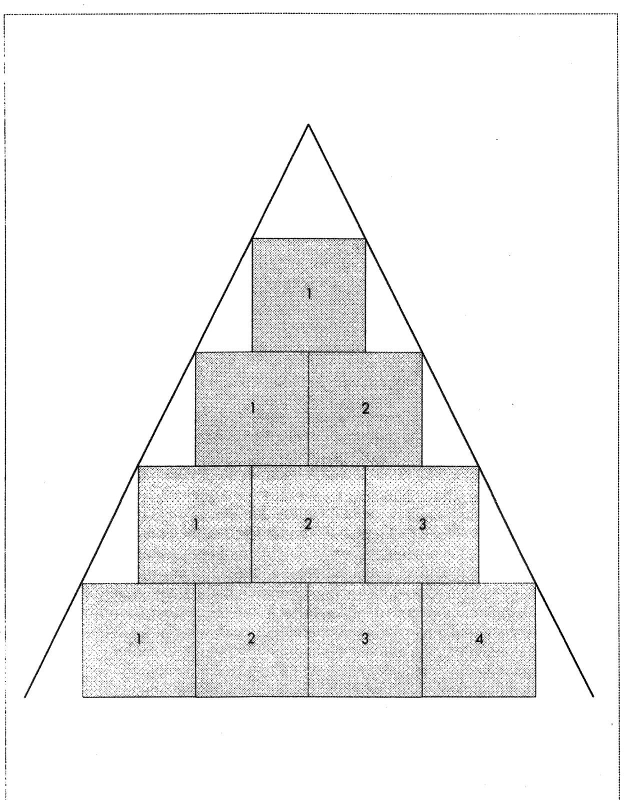

Figure 4: *2 tangent lines to 10 squares.*

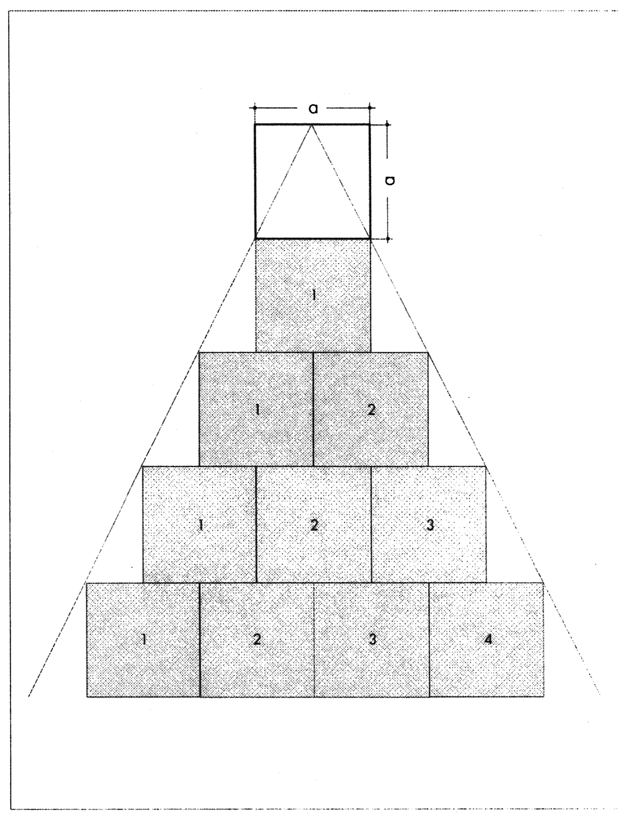

Figure 5 : *Appearance of a new square with upward pointed triangle.*

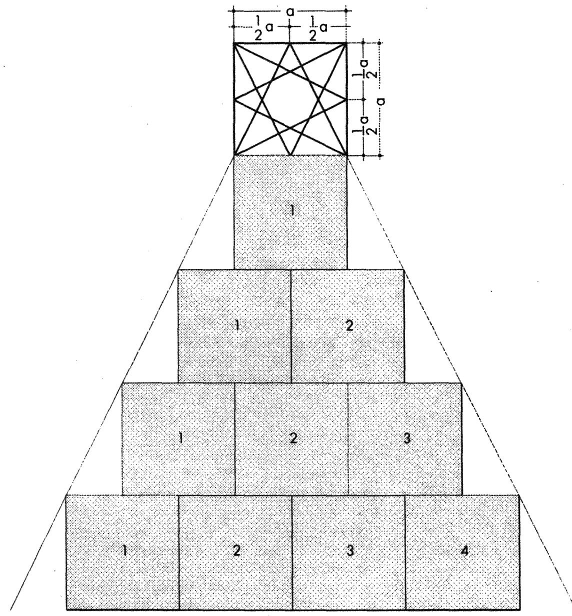

Figure 6 : *Upward, downward and sidewards triangles form an 8-pointed star .*

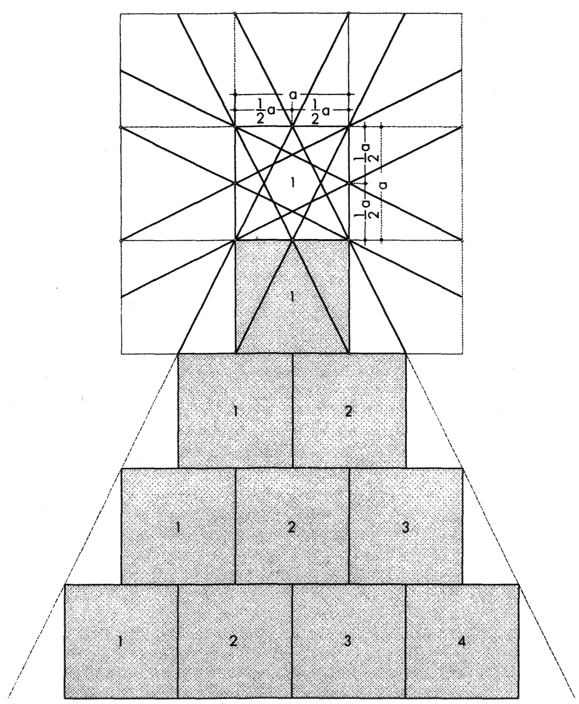

Figure 7 : *The 8-pointed star is expanded to a nine-square grid.*

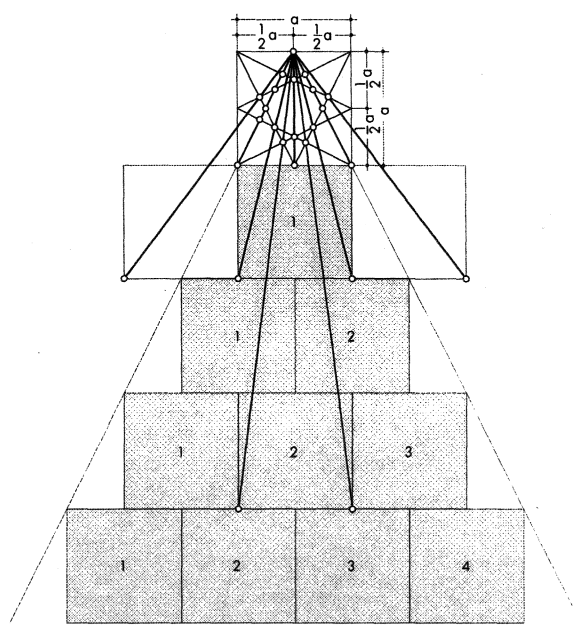

Figure 8 : *Notice how the 8-pointed star is related to the original sequence of squares.*

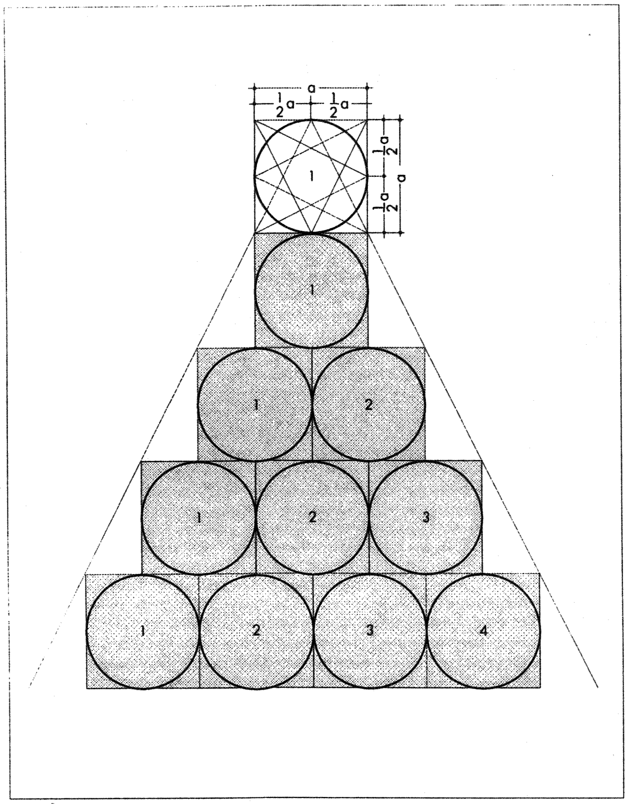

Figure 9 : *Circles are placed within the squares.*

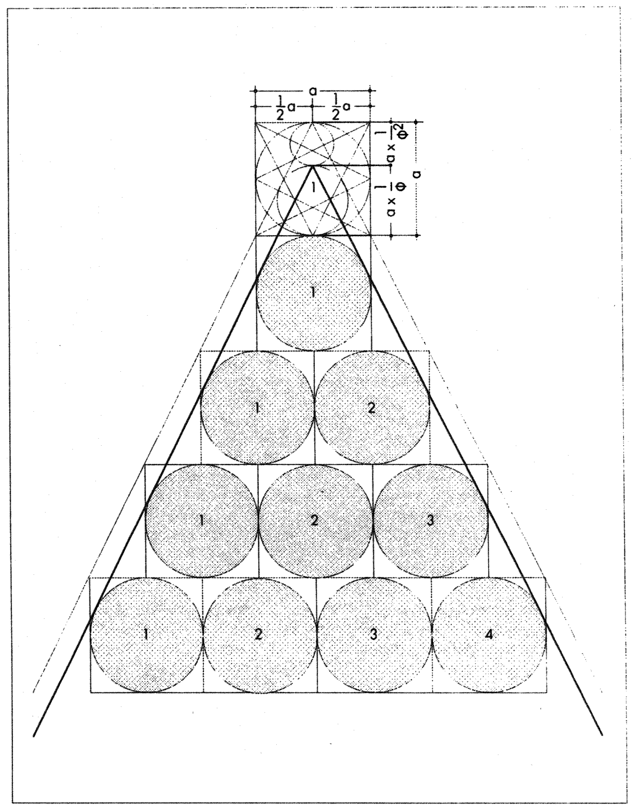

Figure 10 : *A triangle is formed tangent to the circles from which a pair of circles are defined with diameters in the golden proportion.*

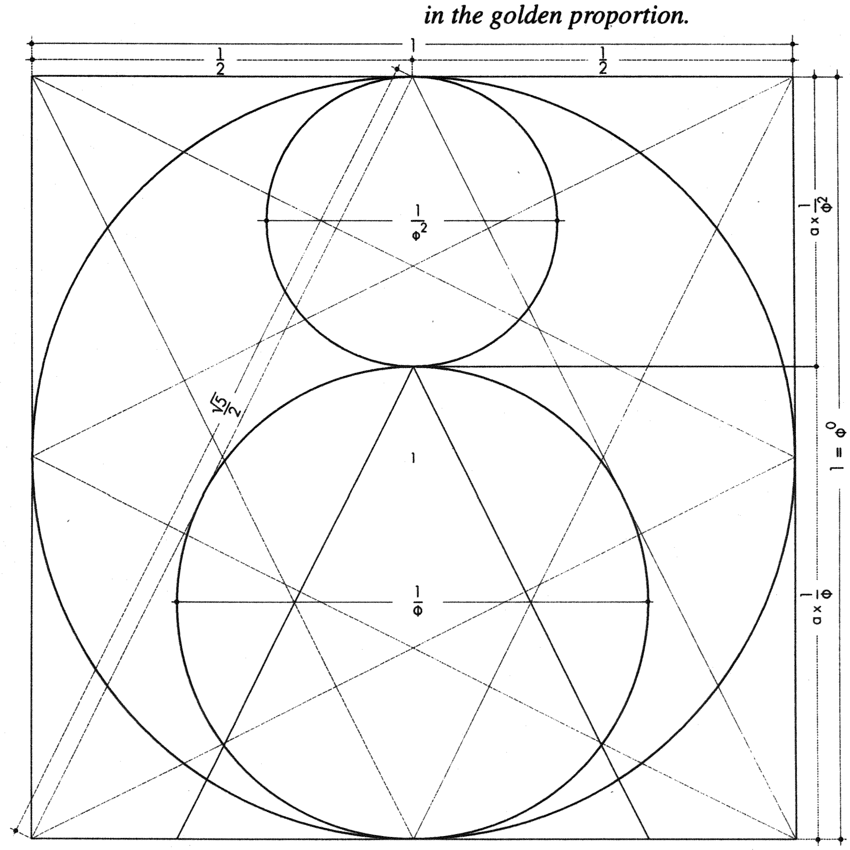

Figure 11 : *The upper square is seen in exploded view.*

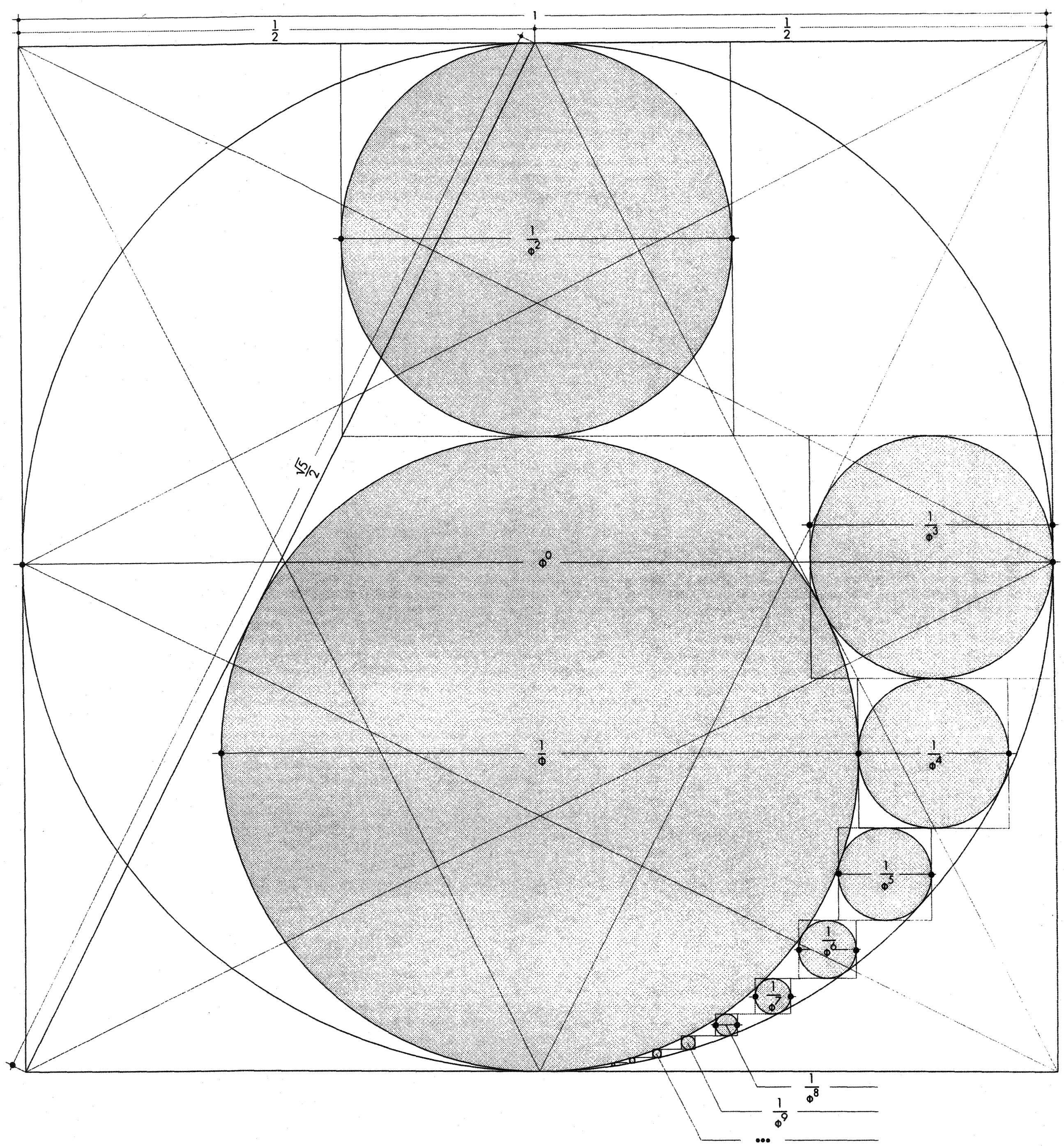

Figure 12 : *A sequence of "kissing" (tangent) circles are created with the negative powers of the golden mean.*

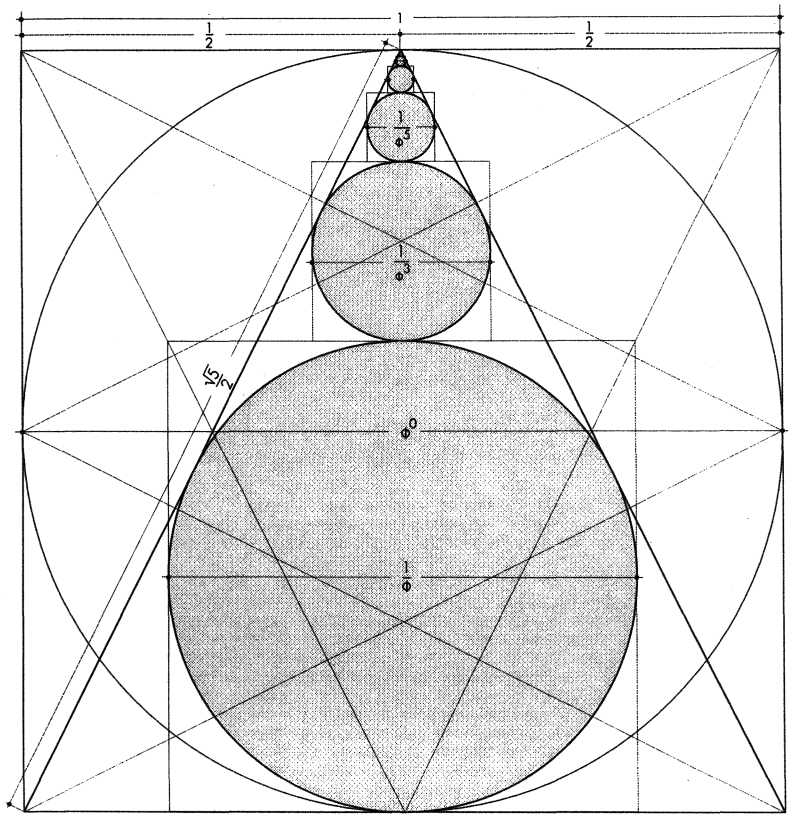

Figure 13 : *A visual proof that the odd negative powers of the golden mean sum to unity [5].*

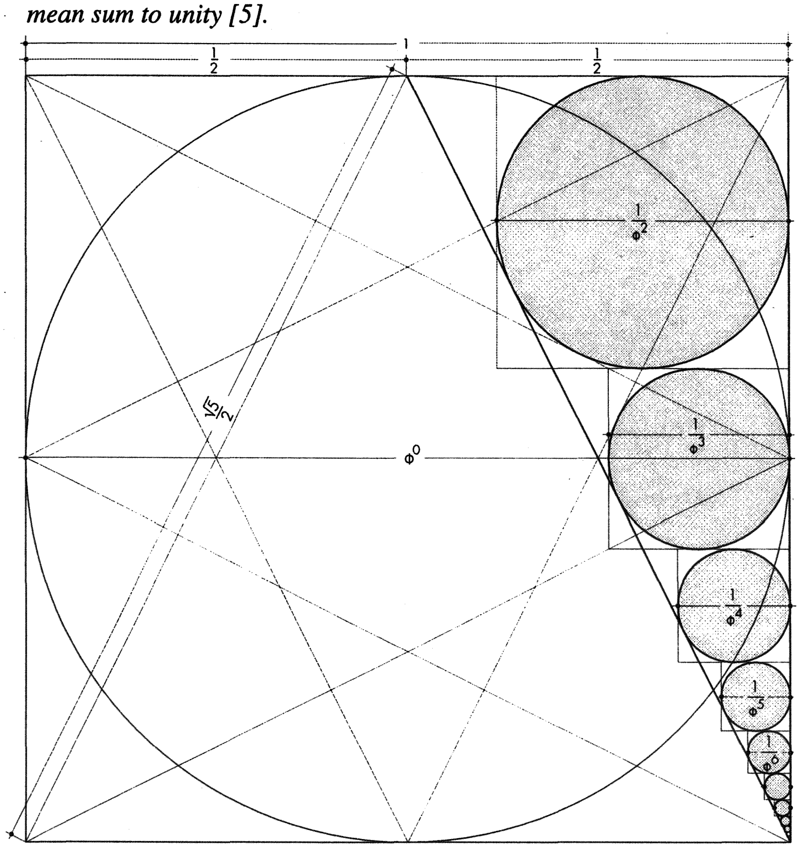

Figure 14 : *All the negative integer powers of the golden mean with the exception of 1/ϕ sum to unity.*

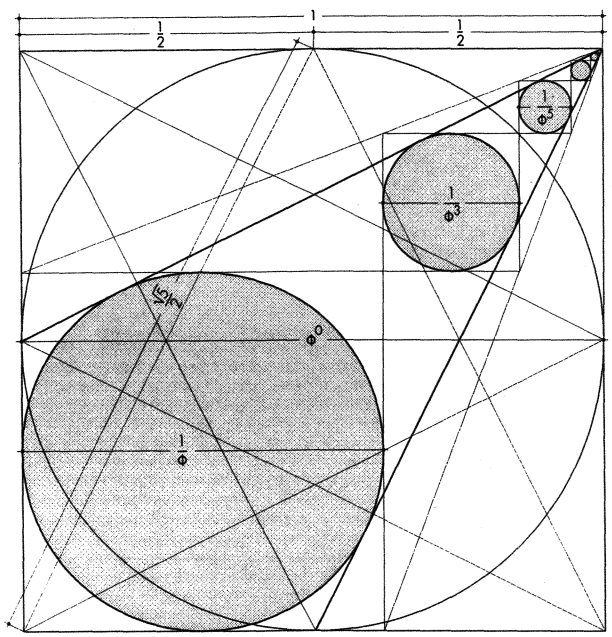

Figure 15 : *Another way to view the odd negative powers of the golden mean as a sequence of circles.*

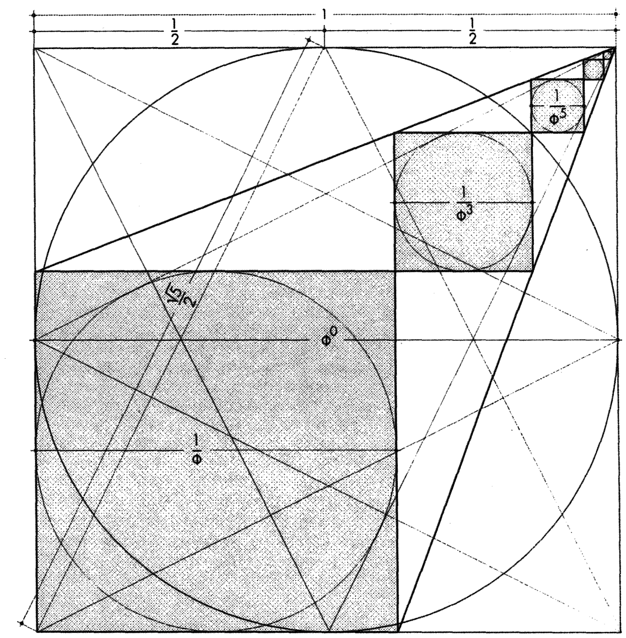

Figure 16 : *They can also be seen as a sequence of squares.*

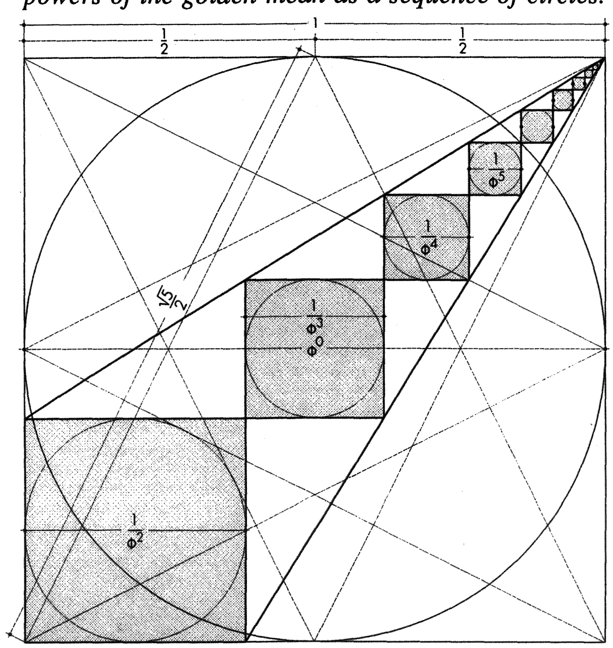

Figure 17 : *The other infinite sequence is seen as geometric series of squares and circles of decreasing size.*

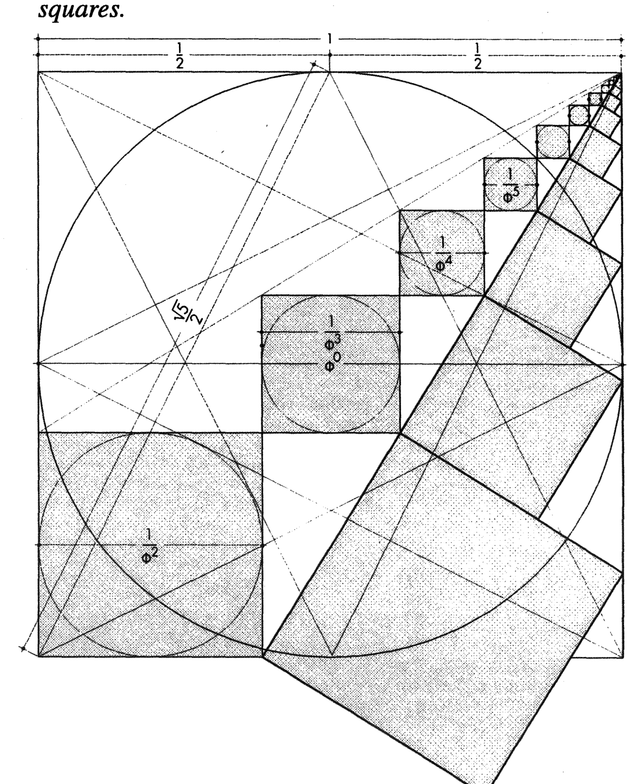

Figure 18 : *The Pythagorean theorem is expressed by this sequence of squares. Notice how a sequence of vertices of the squares upon the hypotenuse lie against the right edge of the framing square.*

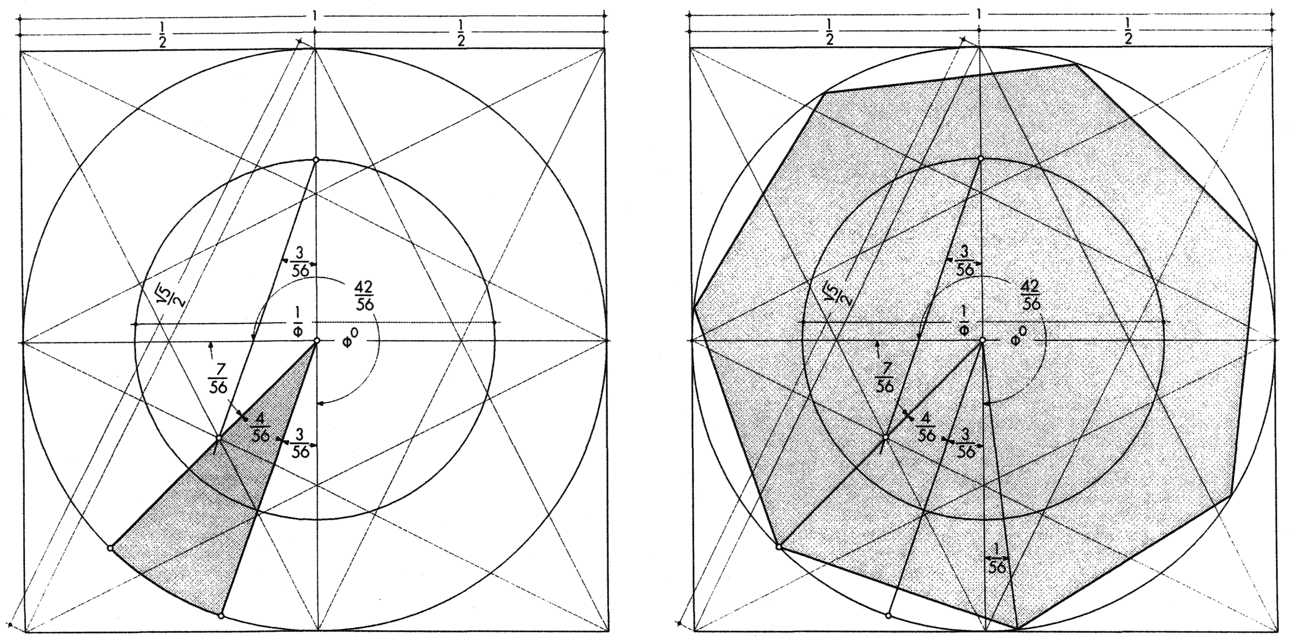

Figure 19 and 20: *An approximate compass and straight-edge construction of the angle of 3/56 x 360 degrees can be related to the golden mean. I found the error to be 0.12 %. Since 7/56 x 360 degrees equals 45 deg. this means that the circle can be subdivided by compass and straight-edge into 56 equal angles to close approximation. It should be noted that the Aubrey circle at Stonehenge has 56 equally placed stones [6]. The 56 subdivisions enable a heptagon to be constructed with compass and straight-edge to within 0.73 % error.*

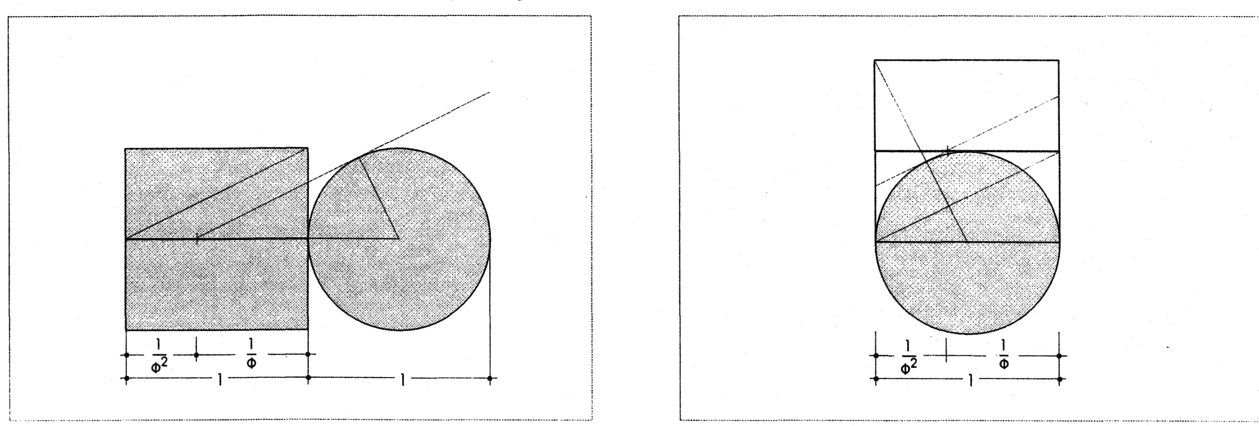

Figure 21 and 22 : *As a result of these findings I have come upon two new constructions of the golden mean based on the relationship between the circle and the square.*

Acknowledgment

I would like to thank Professor Jay Kappraff who showed great interest in my search and not only encouraged me to present part of the findings in this paper but helped in its editing.

Bibliography

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