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Mathematical Connections
in Art, Music, and Science

Number Series as an Expression Model

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Abstract

This work is situated within the scope of the emerging complexity sciences and tries to bridge scientific and artistic intentions for understanding complexity by doing experiments. To this end, we propose the use of simple, "minimal" models coupled with expression functions that translate model results to visual elements or properties. Number series appear to be good candidates for this type of experiment, because they are simple, inflexible and infinite. Expression functions on the other hand may be arbitrary and subjective. Our case study is based on the well-known Fibonacci series and shows that, by constraining some aspect of the visual form, an expression function may translate the number series to a complex visual form. Various expression functions are investigated and their results exemplified by selected images. Several dimensions of future work are also briefly outlined.

1. Introduction

The major concern of the complexity sciences is how simple rules can give rise to complex phenomena and, inversely, how a complex phenomenon may be explained by or "encoded" in a simple rule [8][7]. On the other hand, artists have been traditionally concerned with expressing simple ideas or feelings with complex visual forms and, inversely, with understanding how complex realities can be represented with and express simple, abstract rules [6][9][10][3]. Thus, both scientists and artists often depart from the position that reality is in essence simple but possesses an infinite potential for expression and representation.

The goal of our research is to bridge the intentions of scientists and artists in understanding complexity by experimenting, rather than by using arbitrary complex models to create interesting, complex or aesthetically pleasing visual forms [1][2]. To this end, we start with "minimal" expression models and we try to explore and classify the range of visual forms that they may give rise to. While this is scientifically correct and intriguing, it is not artistically relevant unless we provide a means for artists to express themselves in unique ways. In scientific terms, we should therefore construct a framework that integrates the chosen ***expression model*** (the abstract function or model behind the scenes) together with an ***expression function*** (a translation to a visual form) that may be defined by an artist at will. This expression function will act directly on the structural elements of the visual forms and will translate the results taken with the chosen model to values for those structural elements.

Note that the term "visual form" is used in a general everyday sense that captures all features of structure, color, manner or style, etc. Those features or elements should be explicitly defined and

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represented in a manner with as little arbitrariness and subjectivity as possible. This is in contrast with the actual expression function from model to visual form, where every freedom is allowed and encouraged.

One minimal expression model is a ***number series***, because it is simple (the whole series is usually represented with a single simple mathematical expression), inflexible (once started, it cannot be changed) and infinite (it never ends). The idea is that an arbitrary expression function can produce a wealth of interesting visual forms out of just such a series of numbers. What we seek are simple functions that can produce complex forms out of a number series. Of all available or imaginable number series, we chose the Fibonacci series, because :

- It is a series often encountered in nature and is said to possess aesthetic properties, while being mathematically intriguing as well [5].
- It has been already extensively used by at least one artist, the Italian Mario Merz [4].

Because such a series rises infinitely, it might at first glance seem uninteresting to a spectator to present consecutive values in a row (Mario Merz, "Fibonacci", 1975, and "Fountain", 1978). The next step in complexity has been to translate those numbers to a physical measure (Mario Merz, "Igloo Fibonacci", 1970, & "Fibonacci Drawing", 1977). But again, the measure (instead of the number) rises infinitely. What we would like is, on the one hand, to constrain the system so as to be able to use bounded resources (such as the area of a –finite– painting canvas) and on the other hand to produce infinite complexity despite constraints. We will see in the following sections that constraining the system allows us to do exactly this : produce potentially infinite complexity within limits. *Thus constraining ourselves allows us to become expressive.*

In a longer term perspective, we are seeking ways to express simple rules in aesthetically pleasing or innovative visual forms, so as to be abstract in essence but demonstrate expressive power and/or potential.

2. Expression 1 : Structural functions

The Fibonacci series was defined in the 13th century by Italian mathematician Leonardo Fibonacci as the series of numbers such that every number beyond the 2nd in the series is the sum of the last two : `F(n+2) = F(n+1) + F(n)`. Letting `F(1) = F(2) = 1`, we obtain the series

`1,1,2,3,5,8,13,21,34,55,...`

We obtain different series if we let `F(1)` and `F(2)` take other values, for instance we obtain the Lucas series if we let `F(1) = 1` and `F(2) = 3`. The Fibonacci series possesses some interesting or even impressive properties, for example the ratio `F(n+1)/F(n)` converges to the value 1.6180339... for n going to infinity.

We may bound the values given by the Fibonacci series within an interval [0,d], provided that we find a translation function `f(n):F(n)` $\rightarrow$ `[0,d]`. This translation function is actually the ***expression*** function that expresses and represents a Fibonacci number in a different finite medium, in our case the bounded interval [0,d].

One such expression function is the ***reverberation function*** that translates `F(n)` so that the goal point's distance from the start point equals `F(n)`. If the goal point falls outside the limits of [0,d], the function reverberates in the other direction, and it does so recursively in either direction (from the 0 or d end) until the goal point falls within [0,d]. The new start point is now the goal point and the expression process continues with the next number in the series `F(n+1)`. This process is illustrated in Figure 1.

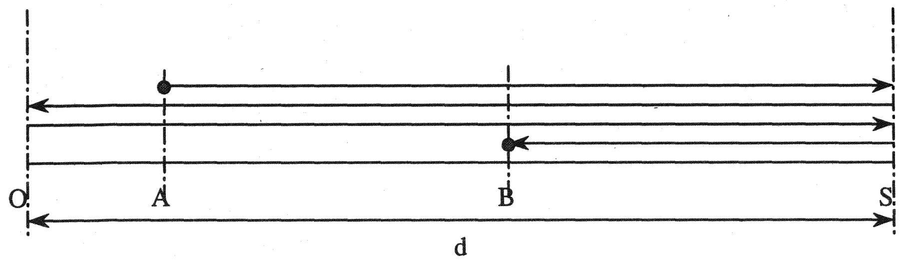

Figure 1: *Reverberation function : With A as start point, the next point is B, where (AS)+2d+(SB)=F(n), n being the current Fibonacci index.*

The reverberation function produces points on the interval [0,d] that very quickly appear to be randomly placed. Furthermore, often enough new points fall on already existing points, so that the process soon slows down in its progress. The process can be started from any start point on the interval [0,d] with any initial index within the Fibonacci series, for instance it could start from index n=7, that is with `F(7)=13`. A sample image is shown in Figure 3(top).

The reverberation function may also be applied to a 2-dimensional area of size $[d_x,d_y]$, with the x- and y- values each following its own Fibonacci dynamics with its own parameters (start value and initial Fibonacci index). In this case, the process will produce points in 2D space that may be connected by line segments, and thus form a linear drawing (Figure 3 middle left). Note that if both x- and y- values start from the same value with the same initial Fibonacci index, the resulting line drawing will be a subset of the diagonal because x(n) will equal y(n), for every n.

The same reverberation function may be applied to the three color components (red, green and blue components), because their values are inherently limited (they usually range from 0 to 255). As is the case with x- and y- values of a point in space, the r-, g- and b- values will have a start value (from 0 to 255) and an initial Fibonacci index. Note that if all three r-, g- and b- values start from the same value with the same initial Fibonacci index, the resulting color will be a level of gray because r(n) will equal g(n) and b(n), for every n. Such colored drawings are shown in Figure 3 (middle right), as well as in Figures 4 to 7.

The above constitute an indication that the resulting images will be more interesting, albeit not information richer, if there is ***diversity*** in the values of similar parameters, for example in the initial Fibonacci indexes for the x- and y- values.

Finally, we can apply the reverberation functions as above to filled shapes instead of points or lines : in the 1D case, we will connect consecutive points with colored bands, instead of drawing colored points, while in the 2D case we will draw filled colored rectangles instead of drawing colored lines (those lines are diagonals of the corresponding rectangles). Colored bands (Figure 3 bottom) or rectangles (Figure 4) may fall on top of previously drawn ones, so that the ***drawing history*** may be hidden in the resulting image.

One can imagine many expression functions that will translate a Fibonacci number within a linear interval or a rectangular region. Another possibility, apart from the reverberation function, might be, for instance, the ***toroidal function***, which assumes that the interval or region is toroidal, i.e., one of its ends is next to the other, so that when a value exits from the right side it automatically re-enters from the left side. This process is illustrated in Figure 2, while Figure 5 presents some visual results produced by this function.

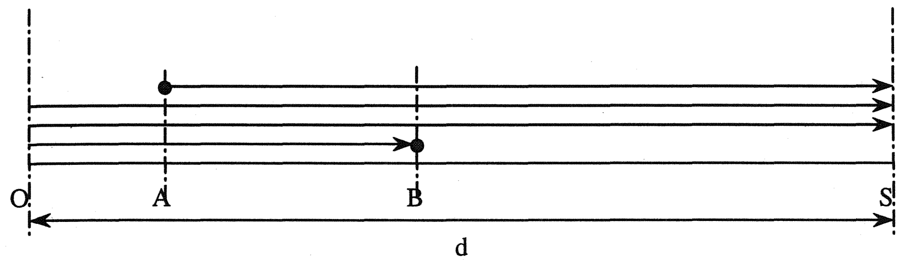

Figure 2: *Toroidal function : With A as start point, the next point is B, where (AS)+2d+(OB)=F(n), n being the current Fibonacci index.*

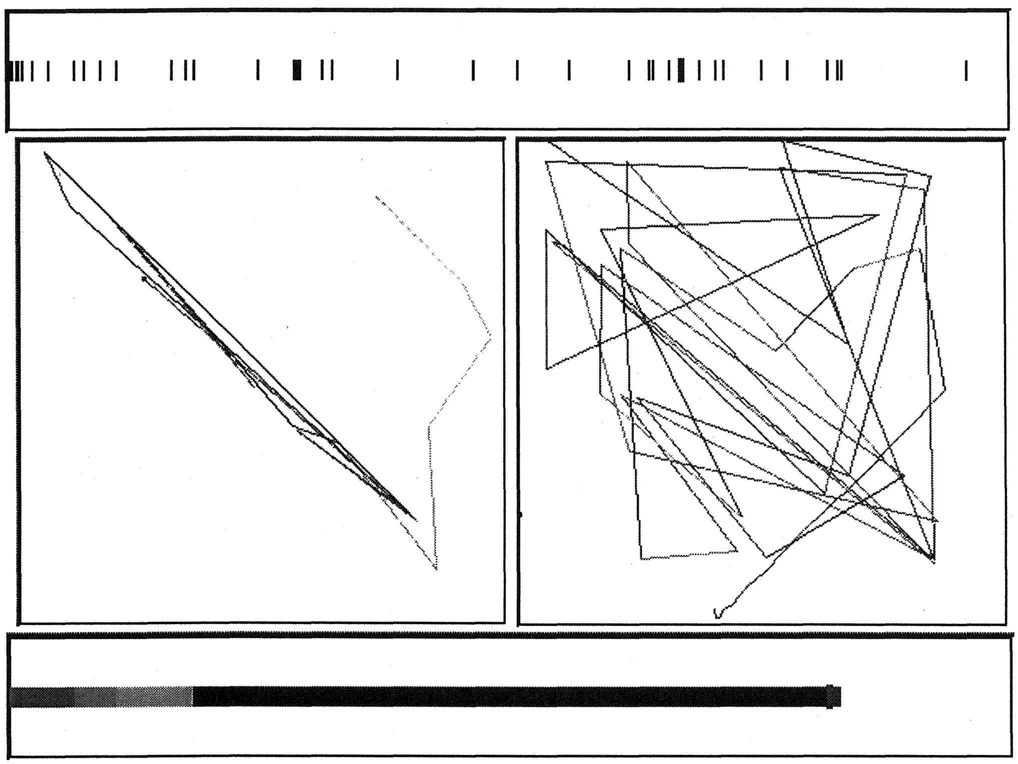

Figure 3: *Visual results of the reverberation function after n=25 or 50 steps. (top) Points in 1D space. For visualization ease, points are shown as short vertical lines. The process starts from the left end and proceeds rightward, reverberating at either end, as necessary. The current point is marked in red color. (middle left and right) Line drawings in 2D space. The x- and y- values proceed rightward and downward respectively, and reverberate as necessary. The middle right drawing uses color reverberation as well. (bottom) Colored bands in 1D space.*

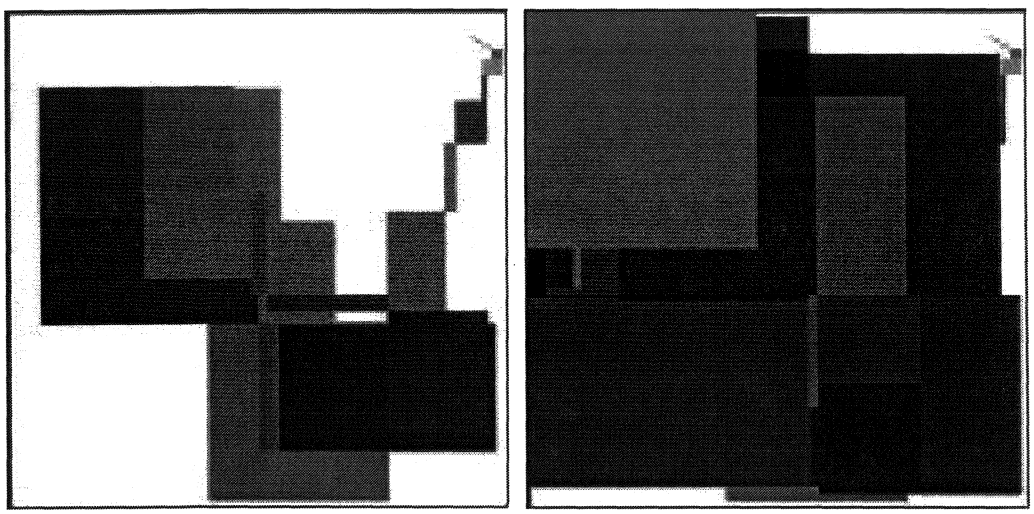

Figure 4: *Colored rectangles in 2D space. The image on the left appears at n=25, while the right one appears at n=50. Note how the small rectangles on the right of the early figure (left) are obscured by others in the late figure (right).*

3. Expression 2 : A behavioral function

Both the functions of the previous section are in essence structural functions that translate a Fibonacci number directly to a structural element of the image without intervention. A more complex case occurs if we allow the expression function to "decide" what to do with the Fibonacci value obtained. For example, in the previous case we observe that the location of the next point is unconstrained within the bounds of the given interval or region. This arrangement often produces large regions that cover older ones, so that the history of the image is lost and color or structure diversity is dramatically reduced (see for example Figure 4 right). One possible solution to this problem is to disallow points from falling far away from their predecessors, by putting some limits to their motion. This is equivalent to introducing a ***behavioral*** function that will act on the pure structural result (as taken in the previous section) and modify it so as to stay within limits :

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If the next point is not within a certain range,
    then the next point is the most recently visited point that
    falls within that range,
    or, if none exists, it is the limit point in the requested
    direction.
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As expected, the new behavioral model produces shorter line segments and smaller rectangles respectively, so that, more colored regions are allowed to coexist in the image, and more diversity is preserved (Figure 6).

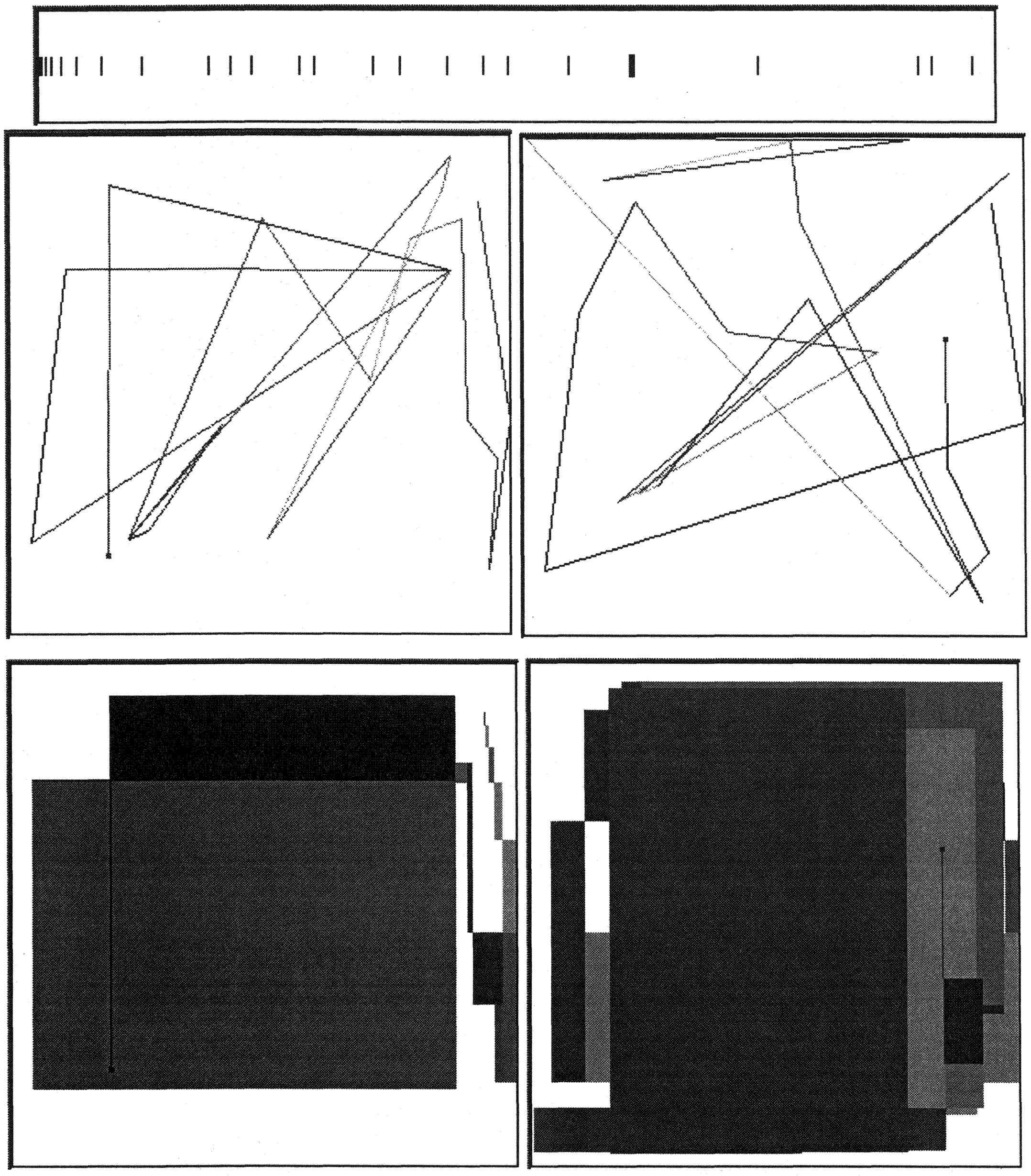

Figure 5: *Visual results of the toroidal function after n=25 steps. (top) Points in 1D space. Compare with Figure 3 (top). (middle) Comparative results in the same initial conditions of the reverberation function (middle left) with the toroidal function (middle right). (bottom) Comparative results in the same initial conditions of the reverberation function (bottom left) with the toroidal function (bottom right).*

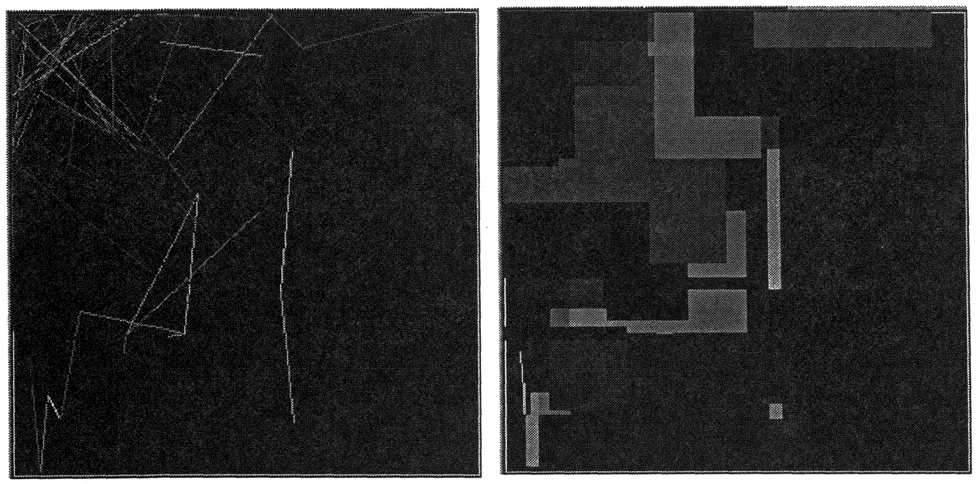

Figure 6: *Series of colored lines or rectangles in 2D space taken with a behavioral model of range 100 (100 pixels is the maximum allowed motion) after 1000 time steps. Both images are taken with a background color other than white. Note that, despite having proceeded in time (n=1000), the images remain far richer than those of Figures 3, 4 and 5.*

Finally, all the above models may be used in the same image by switching from model to model in the course of progress. This way, noticeably more complex images may be produced. For example, the image of Figure 7 was produced by the reverberation function by first using a pure (structural) 2D colored model (rectangles) so as to fill almost the whole region with colors. We then switched to a behavioral 2D colored model (rectangles) so as to produce smaller rectangles, and we finally used the behavioral colored model (lines) so as to produce short colored line segments on top of the existing colored rectangles.

Of course, many more visual features may be defined and manipulated in the same way. Further artistic options may include, for instance, polygonal elements, the number and order of elements, the angle of rotation of geometric shapes, etc.

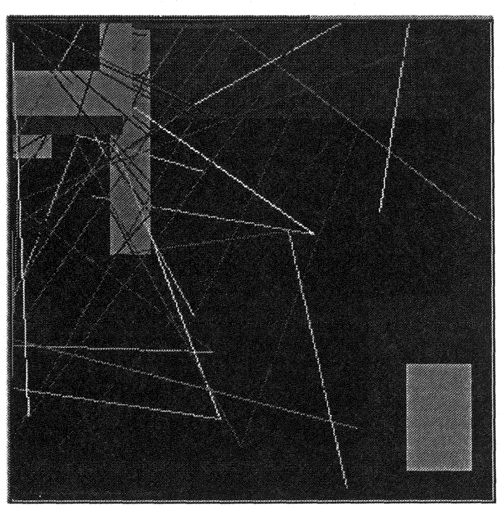

Figure 7: *Image taken by switching at will between models.*

4. Conclusion

We have briefly presented a methodology for studying complex visual forms from the bottom up by choosing a simple expression model and an arbitrary artistic expression function that translates the model's results into visual elements. The key assumption is that some aspect of the visual form has to be constrained so that the potential for infinite complexity arises. Number series are good candidates as expression models and a case study on the Fibonacci series coupled with several expression functions has been presented.

Although our demonstration focused on visual forms (drawings), the methodology may apply equally well to other media of expression. For example, application to serial music is straightforward [13]. Another fruitful direction for research is to try to express number series properties, instead of consecutive values, for example to try to express the fact that the ratio `F(n+1)/F(n)` converges to the value 1.6180339... for n going to infinity. This line of research opens up new perspectives that match both abstract art concerns [11] and early computer art themes that were later abandoned [12].

Acknowledgment

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