BRIDGES
Mathematical Connections
in Art, Music, and Science

Mobius Knitting

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Abstract

Knitting a mobius strip involves mathematical and physical complexities. We discuss several methods for knitting strips, some traditional and some invented by us. Explicit instructions are included for experienced knitters.

Introduction

Sooner or later, all mathematician knitters try to knit mobius strips, and we are no exception. We know several good ways to knit mobius strips, some of which are well-known to knitters. We have invented two new homogeneous methods, both more complicated but more satisfactory than the traditional ones.

Of particular interest are strips with a homogeneous pattern of stitches throughout. More explicitly, we desire strips whose local appearance is the same everywhere on the strip. In particular, we try to avoid strips that possess seams. We will discuss these concepts later in the paper as they arise.

Below we discuss the various methods. We describe some explicit constructions for the reader already familiar with knitting [2] [3]. We ignore most of the physical issues such as tension and friction that determine the exact shape of stitches and fabrics.

Transverse Strips

The most straightforward way to knit a mobius strip is what we call the transverse method, in which the yarn follows a course back and forth across the strip (Figure 3). Each mobius strip diagram depicts a strip that has been cut and unfolded to form a flat rectangle.

Because a mobius strip has only one side, a homogeneous strip must necessarily use a pattern that appears the same on both sides. Basic stockinette (Figure 1) is unsuitable because it has two different sides. For reference, one side is called "knitted" and the other side is called "purled".

Both ribbing and garter (Figure 2) have front-back symmetry, and either can be used to make a homogeneous transverse strip.

The interested knitter can follow the instructions in the appendix to produce a transverse strip. At the end of the construction, the knitter must graft the open loops of the first and last rows of stitches together. Grafting consists of using a large sewing needle to run the yarn through the two sets of loops so as to join them in an identical manner to the way every other row of yarn joins the rows above and below it. We consider this grafting to be a shortcoming of the method. Grafting differs fundamentally from pure knitting in that any knitted piece can be made while pulling yarn from an infinite ball, whereas grafting requires that the yarn be cut and the end worked through the fabric.

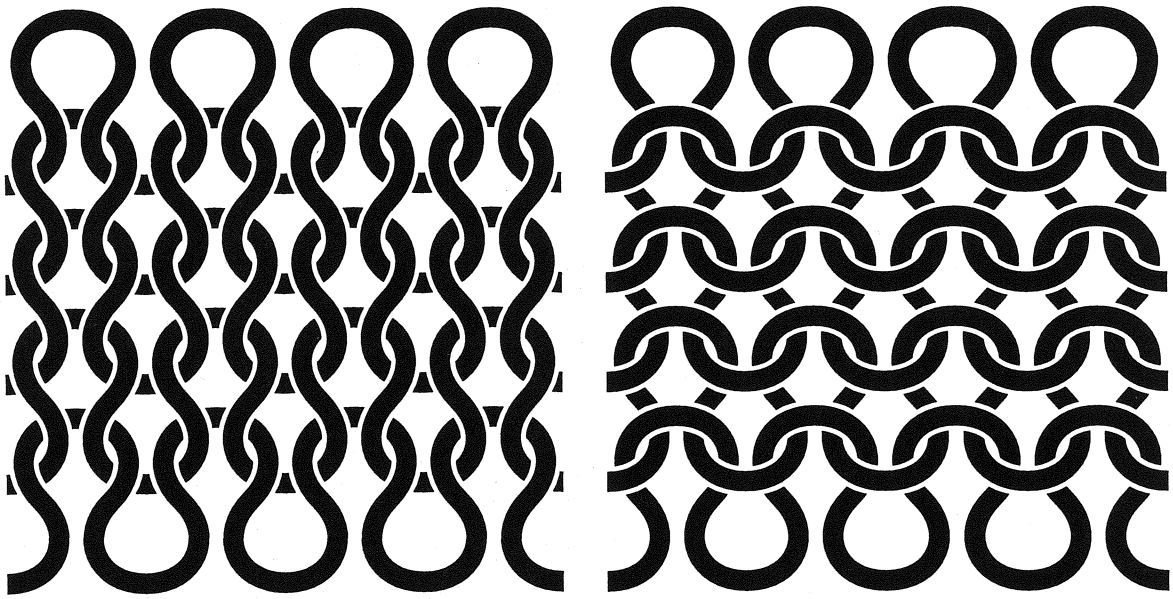


Figure 1: *Stockinette is the most basic knitting pattern. The knitted side (left) and the purled side (right) of stockinette are different, making it unsuitable for seamless mobius strips.*

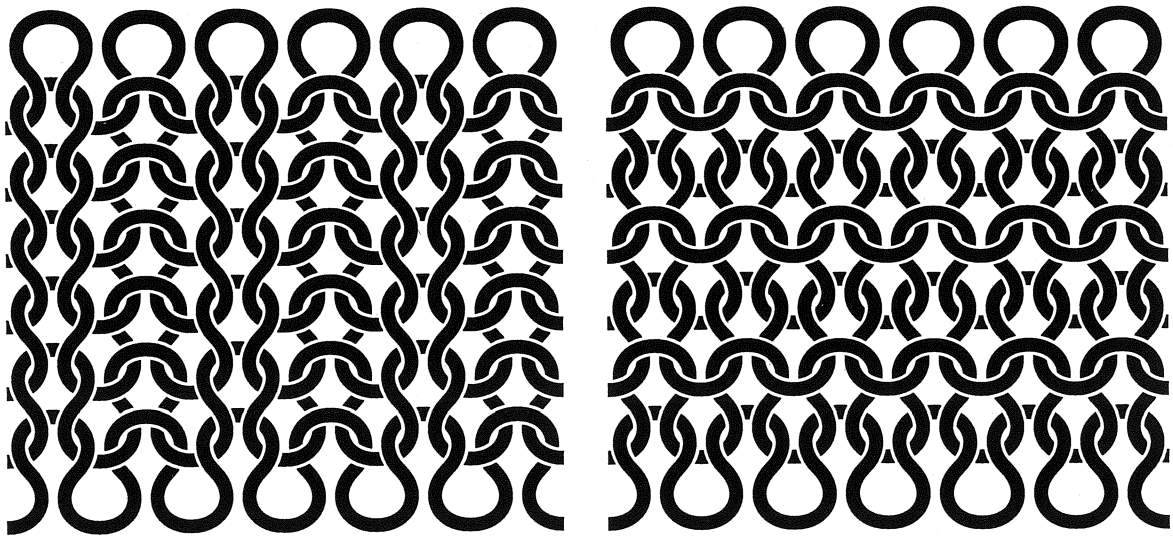


Figure 2: *Two simple patterns: Ribbing (left) is made from alternate columns of knit and purl stitches. Garter (right) is made from alternate rows of knit and purl stitches.*

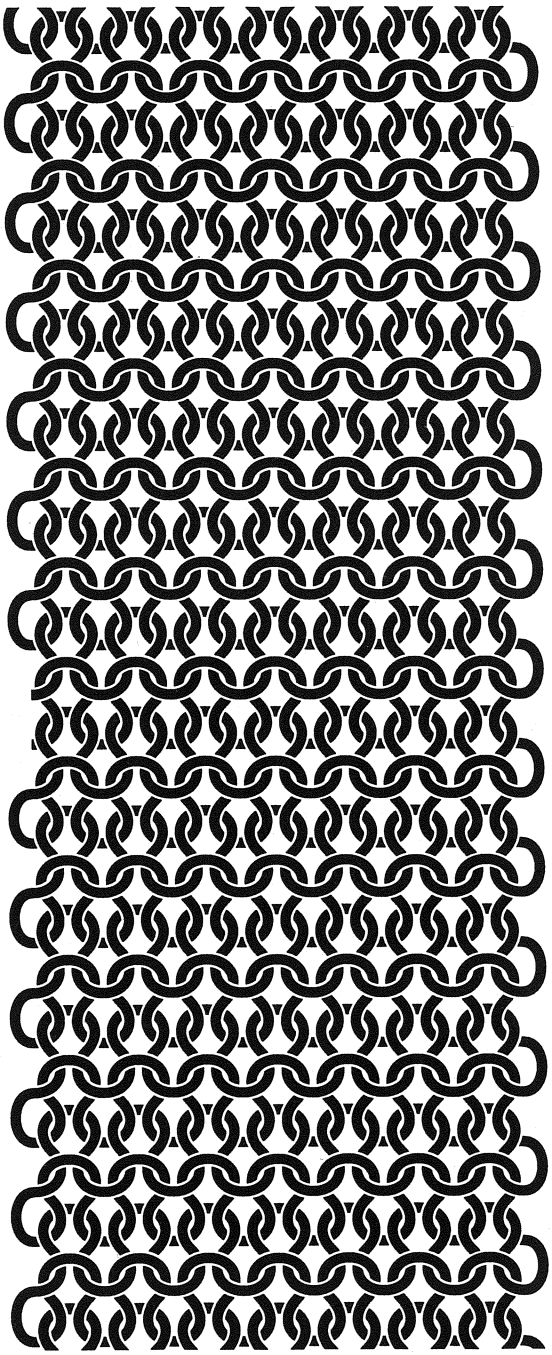


Figure 3: *A transverse mobius strip made from garter. Each mobius strip diagram depicts a strip that has been cut and unfolded to form a flat rectangle.*

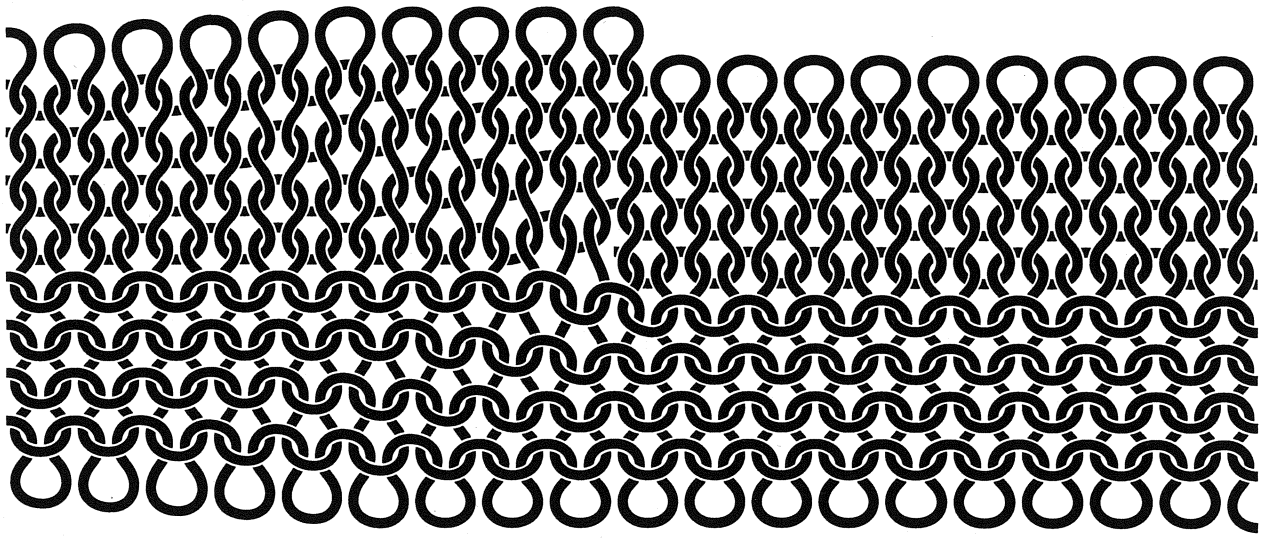


Figure 4: *A helical mobius strip made from stockinette. This consists of a single yarn running from the center of the diagram to the edge.*

Helical Strips

Now we explore a different kind of strip, in which the yarn travels parallel to the boundary. We call such strips helical because the yarn travels in a helical pattern around the strip in an ever-widening path (Figure 4).

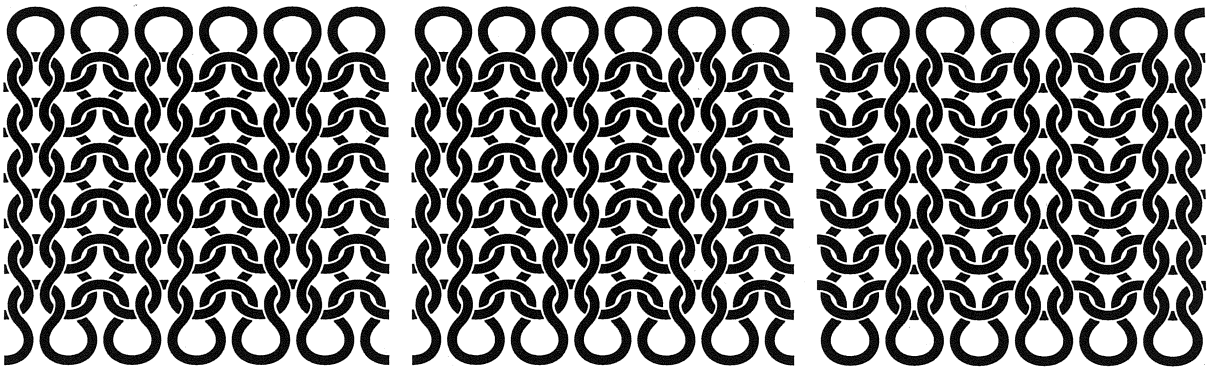
The interested knitter can follow the instructions in the appendix to make a helical strip. A strength of this method is that it only involves knitting; no sewing is necessary in construction. The process is also more interesting than the transverse method because the piece develops as a mobius strip from the start, whereas a transverse strip is only a simple rectangle until the final step.

The helical strip shown in Figure 4 was made with plain stockinette. The pattern of stitches has a very obvious seam, which appears in the figure running horizontally across the middle of the diagram. We use the word "seam" to refer to a line for which the pattern of stitches on one side is not a smooth continuation of the pattern on the other side.

The seam in Figure 4 is inevitable because stockinette does not possess front-back symmetry. However, recall that ribbing and garter do have front-back symmetry, so seamless transverse strips can be made from these patterns. At first, they appear also to be useful in making seamless helical strips. However, more subtle difficulties arise.

A ribbing helical strip also has an obvious seam at the center line. This is inevitable because ribbing is the same on both sides only when flipped left-to-right, not top-to-bottom (Figure 5), making it suitable for seamless transverse strips, but not for seamless helical strips. More precisely, a seamless transverse strip can only be made from a pattern that has rotational symmetry about a vertical axis and a seamless helical strip can only be made from a stitch that has rotational symmetry about a horizontal axis.

Garter stitch has both of these symmetries, but a garter helical strip (Figure 6) has a short transverse seam, which appears in Figure 6 along the vertical line running from the center of the



Obverse | Reverse, flipped left-to-right | Reverse, flipped top-to-bottom

Figure 5: *Ribbing looks the same on both sides when flipped left-to-right, but is different when flipped top-to-bottom. It is therefore suitable for seamless transverse mobius strips but not for seamless helical strips.*

diagram to the top. The pattern to the left of the seam does not align with the pattern on the right. This stems from the fact that garter is a two-row pattern; only every other row repeats. This is a problem in helical knitting because each row is a smooth continuation of the previous row. To produce garter, one must switch between knitting and purling after each cycle around the strip, and these switching points form the seam.

We can avoid this difficulty by using a double helix made from two separate yarns (Figure 7). One yarn is always knitted and the other always purled so that the switching problem does not arise.

We managed to produce a one-yarn seamless helical garter strip by starting in the middle of a yarn and working toward both ends (Figure 8). This strip is very similar to the one described in the previous paragraph except that it consists of a single piece of yarn. Starting in the middle of a yarn is a technique rarely used by knitters, but it satisfies the criteria of pure knittability because the strip can be made without relying on the ends of the yarn.

In all the helical strip diagrams, one can see an area of irregularity around the center point, where there is one more row to the right than to the left, causing the stitches to the left to take a distorted elongated shape. In the double-helix strips, the effect is heightened because there are two extra rows. Nevertheless, because these irregularities are confined to the area around a point rather than a whole line, we do not consider them as troublesome as seams.

Mobius Stitch

Finally, we present a new stitch that we invented expressly for use in helical mobius strips. The mobius stitch (Figure 9) has symmetry about both horizontal and vertical axes. Moreover, every row appears exactly the same. These properties allow the construction of a seamless single-helix mobius strip (Figure 10). The strip has no point irregularities except for the start and end of the yarn. In the appendix we discuss how to make this stitch and how to use it to make a mobius strip.

The challenge to knit a better mobius strip has led us to study the underlying topological principles of knitting. In future work, we plan to make precise mathematical definitions of knittable fabric and study symmetries of such fabrics.

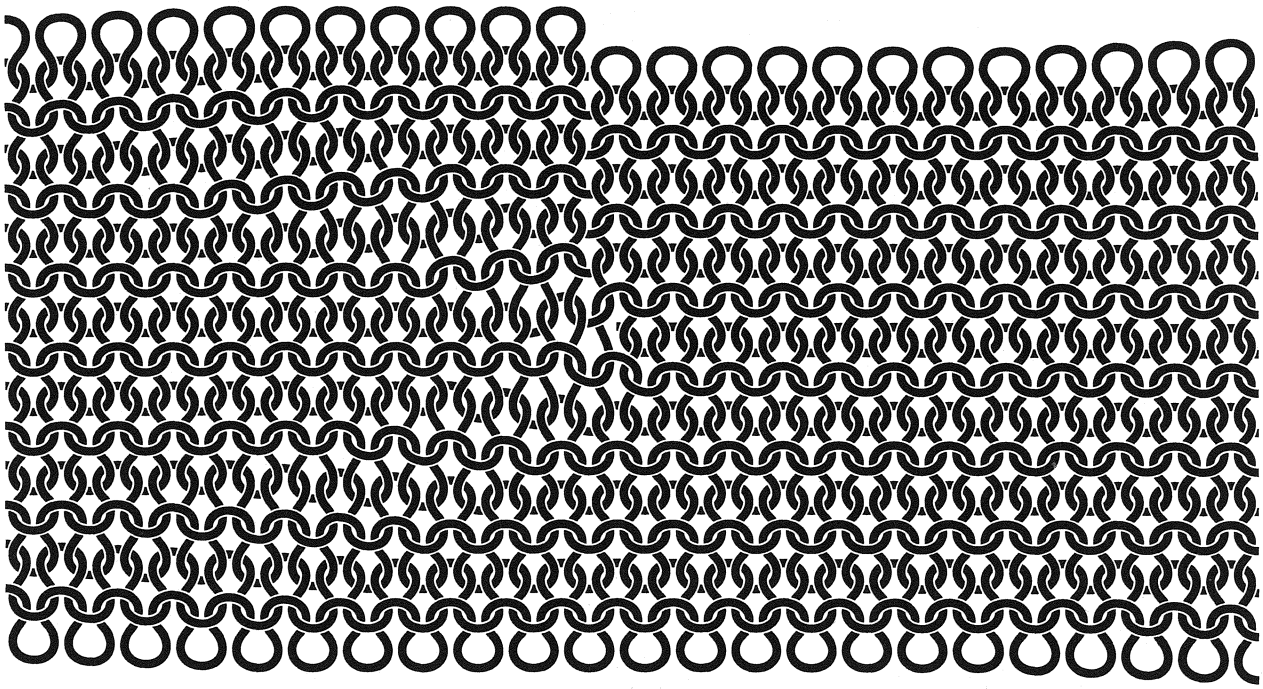


Figure 6: *This helical strip made from garter has a short seam along the vertical line from the center of the diagram to the top. The pattern to one side of the line is not aligned with the pattern on the other side.*

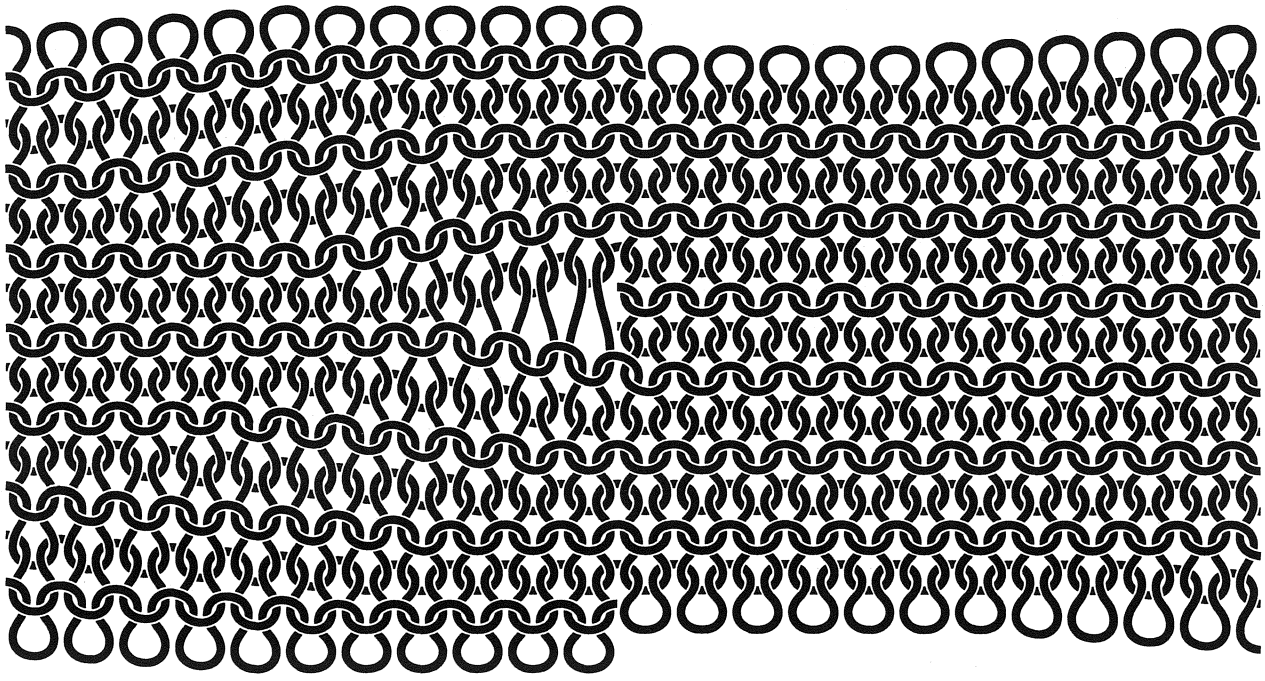


Figure 7: *This helical strip made from garter avoids the seam described in Figure 6 by using two distinct yarns.*

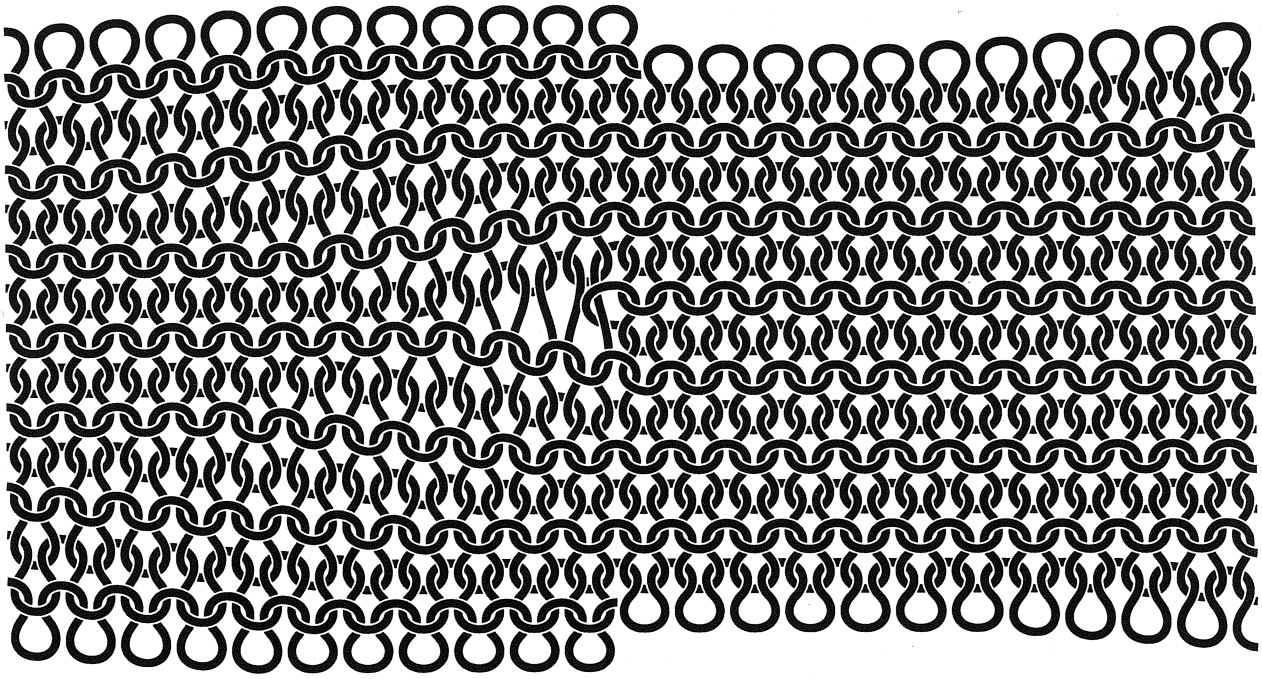


Figure 8: *This helical strip made from garter is seamless and is composed of a single yarn.*

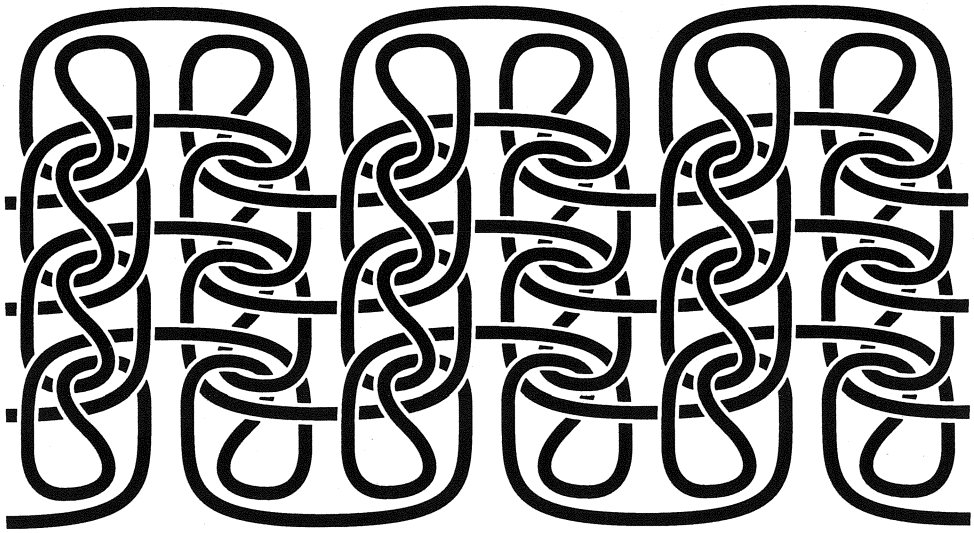


Figure 9: *The mobius stitch is a one-row pattern with symmetry about both horizontal and vertical axes.*

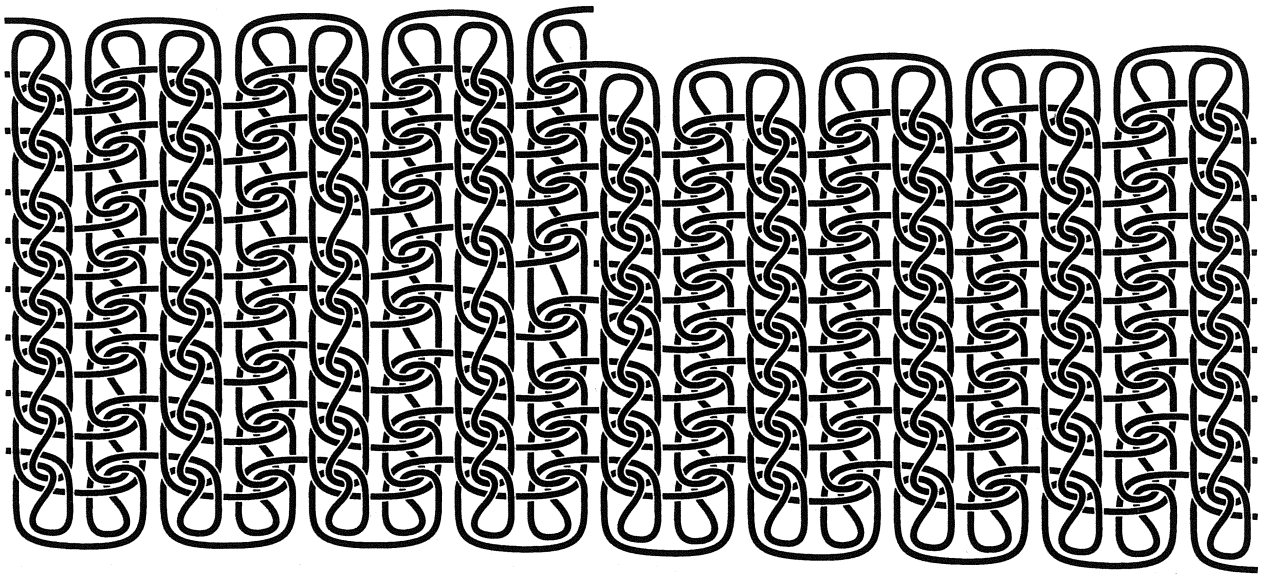


Figure 10: *A helical mobius strip made from mobius stitch has no seams and is composed of a single yarn.*

Appendix

Nomenclature and format for patterns are based on Walker [3]. Unless noted, all slips should be done purlwise, with the yarn in back of the needles.

How to knit a transverse strip. 1. Cast on using the following method. It produces a row just like the last row created by the usual binding off method, so it can unravel. At the end, the last stitch is dropped. To prevent premature unraveling, pull the yarn to enlarge the stitch and loosely tie it.

Yo, * slip 1 back knitwise, yo, k1; repeat from *; end: drop the last stitch.

2. Start knitting many rows to create a tall narrow piece of fabric. If you are using a 2-row pattern such as garter, knit an even number of rows (not counting the cast on row, which will be pulled out).

3. Untie the last stitch cast on and pull out the cast on row. Cut the yarn, give the strip a half twist, and use a yarn needle to graft the ends together. □

See Stanley[2] for detailed instructions for grafting, as well as for other unravelable, or "provisional" casting on methods.

How to knit a helical strip. There are several possible approaches to knitting a helical strip. We formulated the one presented here to be adaptable to different patterns. See Conway *et al.* [1] for a simpler stockinette-only method.

The overall approach is to cast on using an auxiliary yarn, knit the first two rows in a similar manner to normal circular knitting, then pull out the casting-on yarn and knit the third row into

the bottom of the first row.

1. Using a circular needle and the casting on method introduced in the instructions for transverse strips, cast on enough stitches to fill half the needle. This starter row will be pulled out and discarded, so to avoid confusion use a different yarn with a contrasting color to the main yarn.

2. Slide the stitches to the left needle point, switch to the main yarn and knit a row.

3. Slide the stitches to the left needle point. Wrap the needle into a double circle, so that the two needle points and the midpoint of the needle (where the last stitch is located) are all in the vicinity of the same point. Knit a second row.

4. Slide the stitches around to the left needle point. Start pulling out the contrasting yarn and picking up the stitches onto the left needle point. Continue all the way around.

5. Start knitting the third row. The needle is now running along the entire edge of a mobius strip. Continue knitting until the strip is of the desired width. Bind off in the usual manner. □

How to knit the mobius stitch. The mobius stitch instructions use some special terms, which are here defined:

Twist 1st st — slip 1 knitwise, slip 1 back.
Twist 2nd st — slip 1, slip 1 knitwise, slip 2 back.
K2 tog-b dropping 1 — k2 tog-b but slide only the first of the two stitches off the left needle.

The mobius stitch is knit in rows with a multiple of 3 stitches.

All rows — * Twist 2nd st, k2 tog-b dropping 1, yo, p2 tog-b; repeat from *.

How to knit a mobius strip with the mobius stitch.

1. Follow the overall instructions above for a helical strip, but use the following stitches:

Casting on (3n + 2 sts) — Reverse yo left needle, yo, k1, * slip 1 back knitwise, yo, k1, slip 1 back knitwise, yo, yo, k1; repeat from *; end: slip 1 back knitwise, yo, k1, drop the stitch just knit and tie it.

Rows 1 & 2 — * Twist 2nd st, k2 tog-b dropping 1, yo, p2 tog-b; repeat from *; end: twist 2nd, k2 tog-b, yo.

Row 3 — Twist 1st st, p2 tog-b, * twist 2nd st, k2 tog-b dropping 1, yo, p2 tog-b; repeat from *.

2. Once row three is started, you have merged the top and bottom edges to form one continuous mobius strip edge with 2n + 1 sets of three stitches. Keep knitting the 3 stitch pattern until the strip is of the desired width. Bind off in the usual manner, passing each stitch over the next stitch, but don't do yos, and after each purl stitch twist it to the left (slip 1 back knitwise, slip 1). □

To pick up stitches correctly, you should experiment first with slowly dropping stitches of the mobius pattern off the needle and observing how to put them back on correctly. Also note that when the stitches have just had the casting-on pulled out of them, the purl stitches are rotated quite a bit to the left. You must rotate them more than 180 degrees to the right (counterclockwise as viewed from above) to put them on the needle correctly.

For best results, we recommend shortening the stiff ends of your circular needle, which makes it easier to work with when wrapped twice around a small circle. If you do not have a modified needle, it may be easier to do the picking up of stitches on a double-pointed needle and transfer them to the circular needle every few stitches.

References

[1] J. Conway, P. Doyle, J. Gilman, W. Thurston, *Geometry and the Imagination*, unpublished notes from a summer workshop at the Geometry Center, Minneapolis, 1991. (Available at: `http://www.geom.umn.edu/docs/education/institute91`.)

[2] M. Stanley, *Reader's Digest Knitter's Handbook*, Reader's Digest, Pleasantville, New York, 1993. Stoughton, London, 1985.

[3] B. Walker, *A Treasury of Knitting Patterns*, Charles Scribner's Sons, New York, 1967.