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Mathematical Connections
in Art, Music, and Science

Reading Numbers as a Metaphor of the Universe

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Let us consider the sequence 1, 11, 21, 1211, 111221, ... , where each term is the reading (by a machine) of the preceding one: 11 means "once one", i.e., the reading of the first term; 21 means "twice one", i.e., the reading of the second term; 1211 means "once two once one", i.e., the reading of the third term, etc. Given a positive integer a, its successive readings r(a), r(r(a)), ... leads to an infinite sequence s(a) of positive integers. It is proved in [2] that the only positive integer a such that r(a)=a is a"; for any a different from 22, its successive readings form an infinite set. Any positive integer can be conceived as a word on the alphabet of digits from 0 to 9. It is said that a is an atom [1], if no decomposition of a¼ is possible, where b and c are both of length different from zero and r(a)-r(b)r(c), r(r(a))=r(r(b))r(r(c)), ... The following results are formulated (with only a sketch of proof) in [1]: There are infinitely many atoms, but only 92 of them are involved in the iterative reading of any positive integer different from 22 (remember that 92 is just the number of the types of chemical elements, i.e., of atoms, existing in nature). Each of these 92 atoms contain exclusively the digits 1, 2, 3; they are the only ones that proliferate beyond any limit (the "malign cells" in the body of positive integers, in contrast with the other digits, which are "benign"). Just as in the Mendeleev periodic table, these 92 atoms are guided by some precise rules. The element of rank n is inside the reading of the element of rank n+1 or equal to this reading; the atom 3 is inside its reading, the atom 13, which in its turn is inside its reading 1113. The "chemical theorem" asserts that the interative readings of any of the 92 atoms are compounds of these atoms; each reading of order sufficiently high of any positive integer other than 22 (the latter is the hydrogen !) involves all 92 atoms. So, these 92 atoms are the universal bricks of the world of positive integers, in respect to their interative readings, just in the same way in which atoms in physics and chemistry are the universal basic bricks of our universe. There is also a "cosmological theorem", asserting the existence of a threshold, the same for any positive integer other than 22, beginning with which a very simple order appears; it is the arithmetic Big-Bang, separating order from disorder, cosmos from chaos.

In all the above considerations, positive integers are supposed to be written in base 10. I asked my colleague Dan Schwarz [3] to check to what extent things are similar when base 10 is replaced by another one. The answer, rigorously proved, is: similar theorems are valid for any base larger than 3; for bases 2 and 3, things are partially different, partially similar.

Mathematics is not only connected to art; it is just art. It accounts, holographically and metaphorically, for the whole universe.

References

[1] J.H. Conway, *The Weird and Wonderful Chemistry of Audioactive Decay.* In T.M. Cover, B. Gopinath (eds), "Open Problems in Communication and Computation". Berlin: Springer, 1987, p. 173-188.
[2] J. Dassow, S. Marcus, G. Paun, *Interative Reading of Numbers: The Ordered Case.* In "Development in Language Theory. At the Crossroad of Mathematics, Computer Science and Biology" (eds. G. Rozenberg, A. Salomaa). Singapore et al.: World Scientific, 1994, p. 157-168.
[3] Dan Schwarz, Personal Communication. Montreal, 1994.