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Mathematical Connections
in Art, Music, and Science

Recognizable Motif Tilings Based on Post-Escher Mathematics

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M.C. Escher was preoccupied for most of his career as an artist with the covering of the Euclidean plane by tiles with recognizable motifs, generally lizards, birds, and the like. In his notebooks, 137 such designs are enumerated, and several of these were used as the bases for some of his best-known finished prints. All of these sets of tiles only fit together one way.

New tilings with recognizable motifs have been designed based on mathematical discoveries made around the time of Escher's death in 1972. In particular, the nonperiodic Penrose and related tiles, and the concept of fractals are employed.

In the case of Penrose and related tiles, the edges of rhombi are replaced with a single line segment in different aspects. First, a pair of rhombi with interior angles $\pi/5$-$4\pi/5$ and $2\pi/5$-$3\pi/5$ is considered. Of the numerous distinct ways in which a line segment can be placed in the eight locations (defining matching rules), some do not tile at all, at least one only allows nonperiodic tilings (the Penrose set P2), and some allow both nonperiodic and periodic tilings. The set P2 allows an infinite number of distinct tilings, and recognizable-motif tiles based on this set have been demonstrated. A set of matching rules which allows both nonperiodic and periodic tilings is used to form tiles with a sea life motif. Generalized Penrose rhombi for $n = 6$ and $n = 14$ (cf. 5) and the Penrose set P1 are also explored and used as templates for multiple-solution recognizable-motif tilings.

The concept of fractals was not well developed or widely known during Escher's lifetime. Even so, several of his prints possess some fractal character. These prints, in which infinite tilings of the Euclidean plane are represented in finite areas have been divided into three categories by Bruno Ernst: square-division prints, spiral prints, and the Coxeter prints, which employ hyperbolic geometry. These are fractal in the sense that they exhibit self-similarity on different scales; however, none of the tiles or perimeters of these prints are fractal. Original recognizable-motif tiling prints have been devised which have different types of fractal character. A design with a fractal perimeter and a non-fractal seal motif is one example. Two designs with fractal tiles have been executed in which singularities are distributed throughout the designs, not just at the perimeters. Finally, a self-replicating fractal tile is demonstrated which has a dragon motif. This tile is the most fractal of the lot in the sense that it is generated by an iterative process. The box counting method to used to compute a fractal dimension for this tile is approximately 1.5.

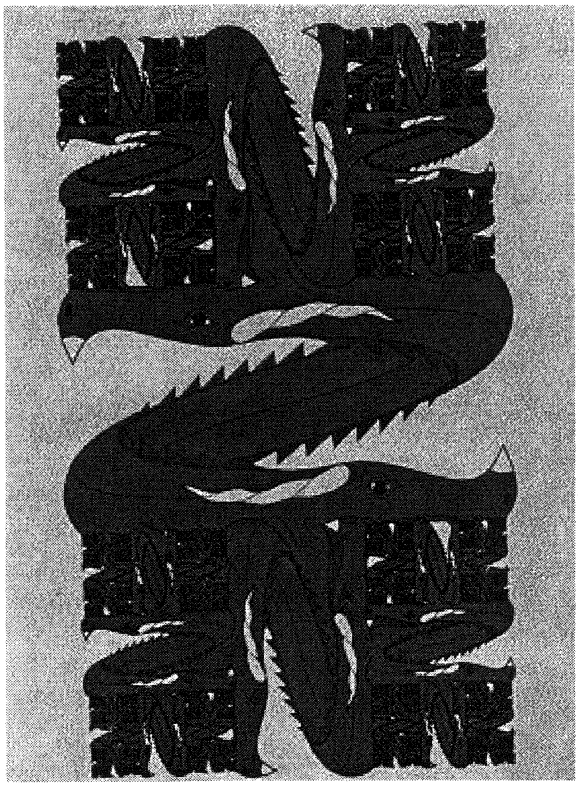

Figure 1: *Fractal Serpents*

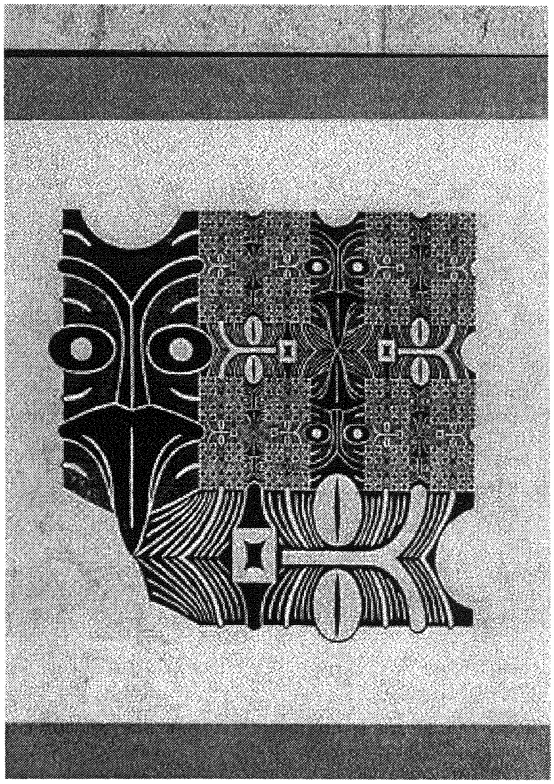

Figure 2: *Fractal Maskes*

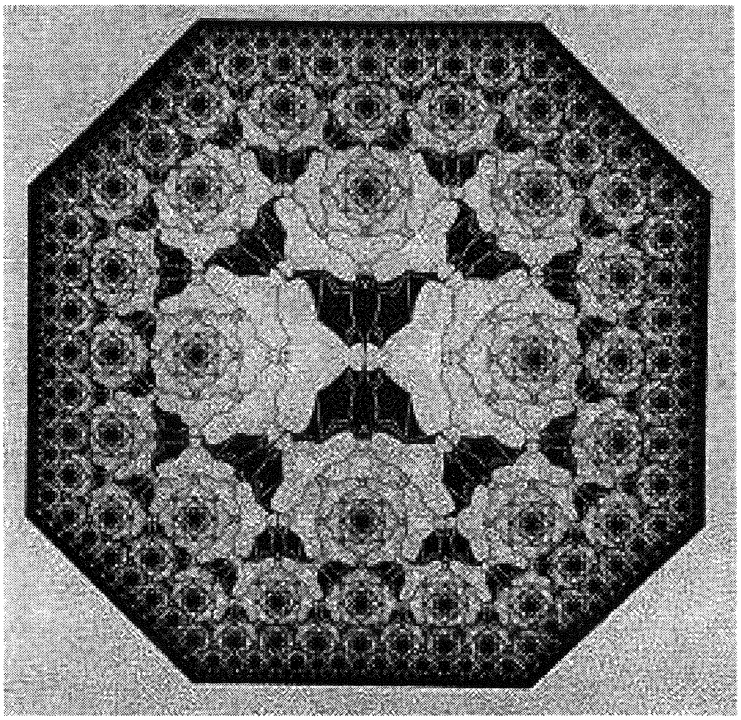

Figure 3: *Bats and Owls*