BRIDGES
Mathematical Connections
in Art, Music, and Science

Merging Paradigms

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Abstract

The author narrates the organic integration of two formerly separate negative curvature design paradigms to form a third metaphorically viewed as a motif cycle of "atomic flowers". This blending of motifs could be regarded as a classic illustration of the aphoristic injunction to "connect, always connect", which also characterizes the Bridges' project of a continuum across disciplines. The author, a non-mathematician, has sought to create a visual aesthetic of mathematical relationships in subtle resonance with the natural world, and in effect an art whose provenance may seem as much natural as cultural, reminding us the dichotomy between them is merely a convention deceiving our place in nature, a deceit only bridged by the delicate lattice of illumination.

Introduction

Since the mid-1990s I have created sculptures in several motifs which differently format the negative Gaussian curvatures found in the Scherk and Costa minimal surfaces (Figures 1 and 2), the latter being our century's unique contribution to the gallery of minimal surfaces and the first to be graphically born in virtual reality through computer programming. Negative or zero curvature surfaces have corollary pairs of curves which radiate in opposed and canceling directions from a central point of symmetry. The familiar equestrian saddle is an easily visualized example, albeit a functional rather than mathematically optimized form. All these sculptures originated, however, as purely intuitive creations from the mind of a non-mathematician. Their relationship to the earlier Scherk and more recently discovered Costa surfaces is consequently independently parallel rather than derivative. There were, in fact, moments fraught with the sudden realization that I had created homeomorphically related forms in my sculptures as a focus of aesthetic attraction. Such beautiful surfaces also appear in the iridescence of soap films where nature's pathway of minimum energy expenditure simultaneously creates the greatest area economy and structural integrity. Similar surfaces which minimize an integral of bending energy, as nature does in elasticity, have been incorporated in automotive CAD programs to convey an elegant aerodynamic advantage to corner curves. The aesthetic temper of nature's economy is, of course, a perennial inspiration, being part of the hopefulness infused in the recurrent cycles of life itself.

So, while the unifying mathematical paradigms for each of the several recent motif configurations in my work are distinct, they are related in their negative curvatures: they differ in the denouement of these curvatures in space. The point being that it is this relatedness which underlies the seamless marriage which is the subject of this discussion; in effect the convergence of formerly distinct motif paradigms to evolve a third. When this happened, I immediately felt the persuasiveness of the floral metaphor for the Eddington atom which was the outcome. Simultaneously, it dawned on me that this had the appeal of something I had until now not found an approach to but long sought -- and continue to seek. The work-in-progress as I write, for instance, seems a more successful approximation that my first effort which appears in Figures 10 and 11 here. All this confirms again how the destination to which years of work have led may be impossible to know until it explicitly arrives as the paradox of a surprise which immediately seems an inevitability, a surprise whose now apparent obviousness as a solution hardly the vaguest premonition anticipated until its moment. That it was being somehow implicitly striven for all along is shown by the relief from tension felt when it is first glimpsed, and to be sure, enough was known for its significance to be instantaneously recognized.

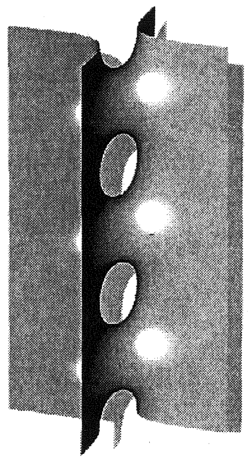

Figure 1: *Scherk's second minimal surface which has equestrian-form saddles (Courtesy of Jim Hoffman).*

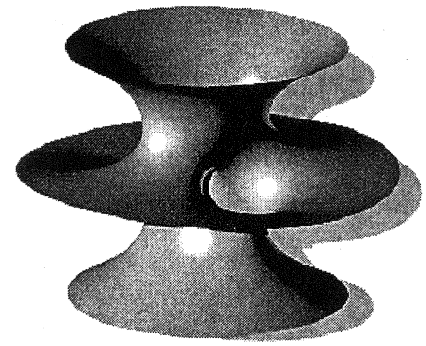

Figure 2: *The Costa minimal surface which can be homeomorphically approximated by toroidally warping Scherk's second minimal surface in the azimuth orientation which closes one of the ends at the center of the torus and locates the end opposite to it at the peripheral mid-riff of the torus (Courtesy of Jim Hoffman)*

Toroidally Warding the Scherk Surfaces: The First Gaussian Paradigm

I've entered the foreground of my subject without sufficient background, though, which I must now turn to. In the first negative curvature paradigm, I warped the uni-directional progression of saddles in the Scherk minimal surfaces into toroidal closures (Figures 3 through 6). The radial axis of these surfaces consists of a imaginary straight line passing through the center of symmetry in each successive saddle. It is precisely the maneuver of bringing this axis into a ring closure which converts the Scherk surface into a Costa homeomorph in formulations of my sculptures with an even-number of saddle modules (Figures 3

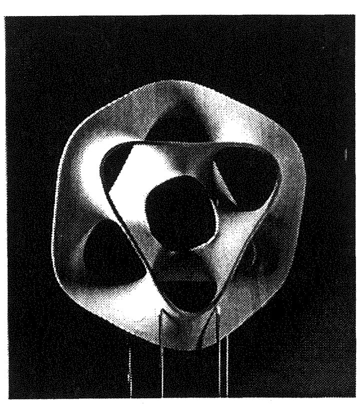

Figure 3: *Hexagonal toroid of equestrian saddles whose retracted ends leave a window open at the center of a topology otherwise a homeomorph of the Costa surface.*

and 4). Such even-numbered formulations require no twist for closure, and in their consequent two-sidedness have an aesthetically taut economy of stasis. Odd-numbered formulations are more dynamic in the twisting vorticism necessary for their toroidal closure and consequent one-sidedness (Figures 5 and 6).

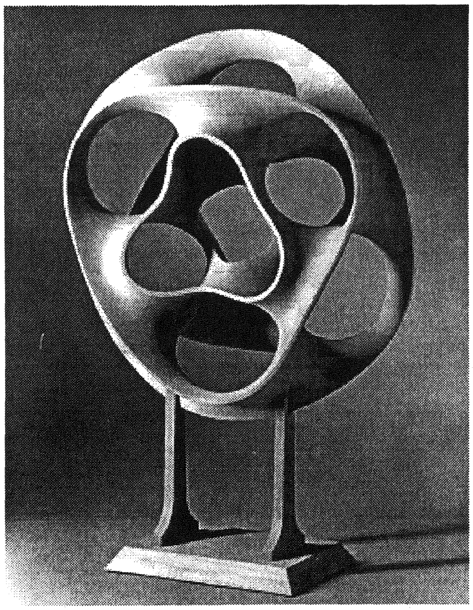

Figure 4: *Hexagonal toroid of higher order "monkey saddles" whose more complex topology nonetheless preserves a mutated affinity with the Costa.*

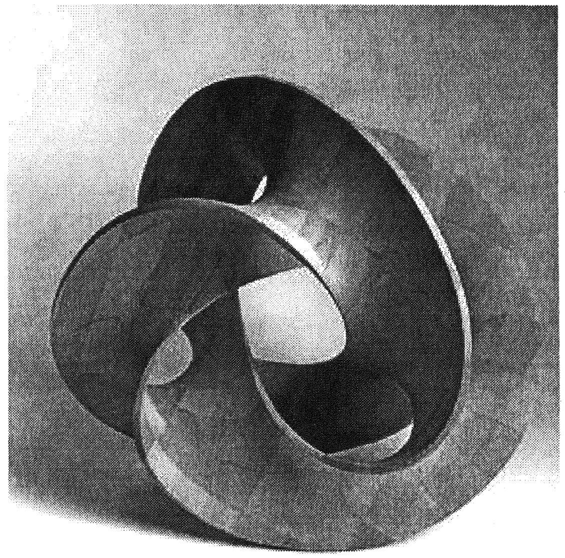

Figure 5: *Non-orientable torus of three equestrian saddles whose trefoil edge pattern is integral to its 270 degree twist parameter.*

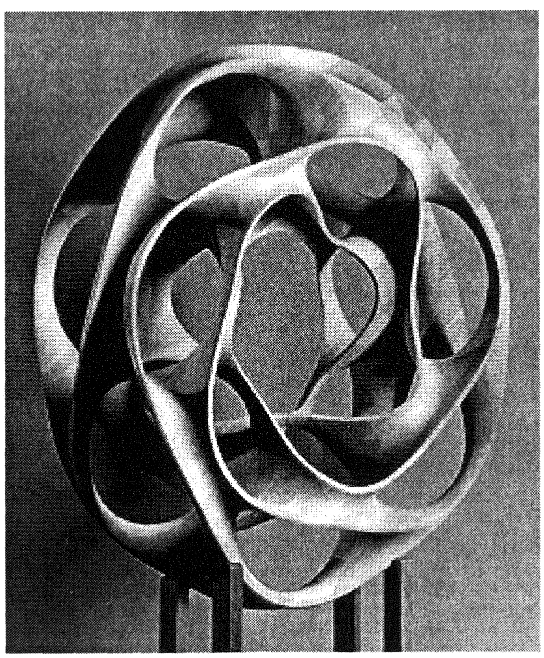

Figure 6: *Non-orientable computer-templated heptagon of quadrupedal saddles.*

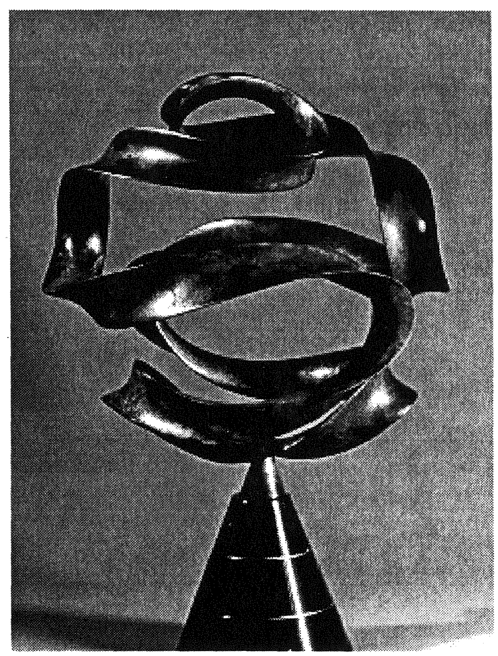

Figure 7: *"Pax Mundi", a sine-ribbon skirting a sphere whose twist parameter maximizes negative curvature*

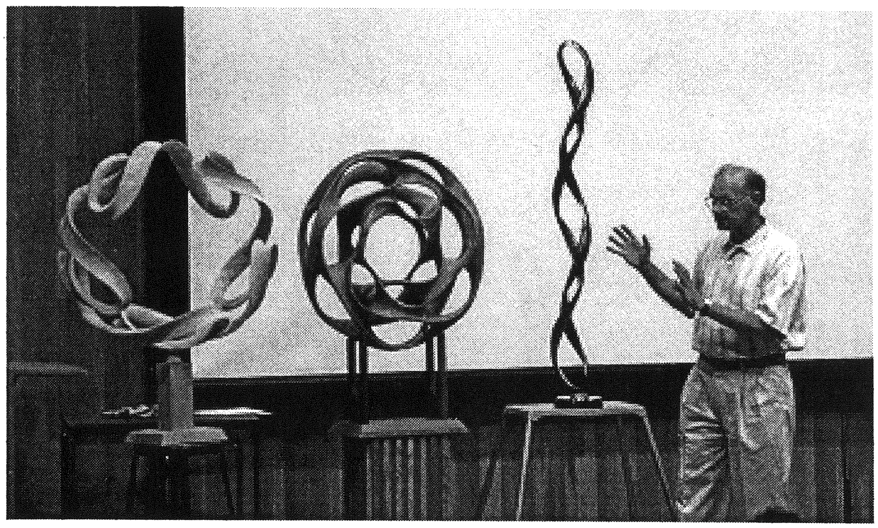

Figure 8: *"Vox Solis" (far left), a negative curvature ribbon deployed on a torus.*

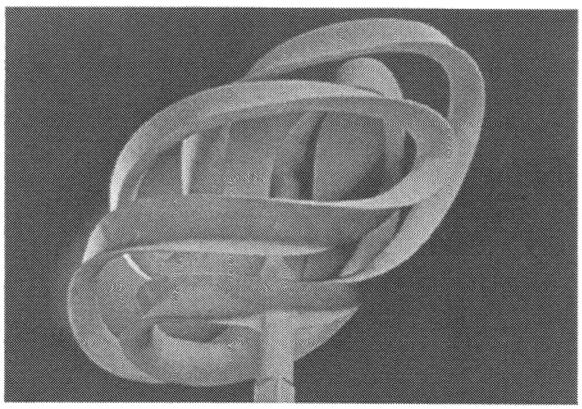

Figure 9: *"Ovum", a negative curvature ribbon loop in a spiral pattern on the surface of an egg geometry in which some segments of its sinusoidal path engulf others.*

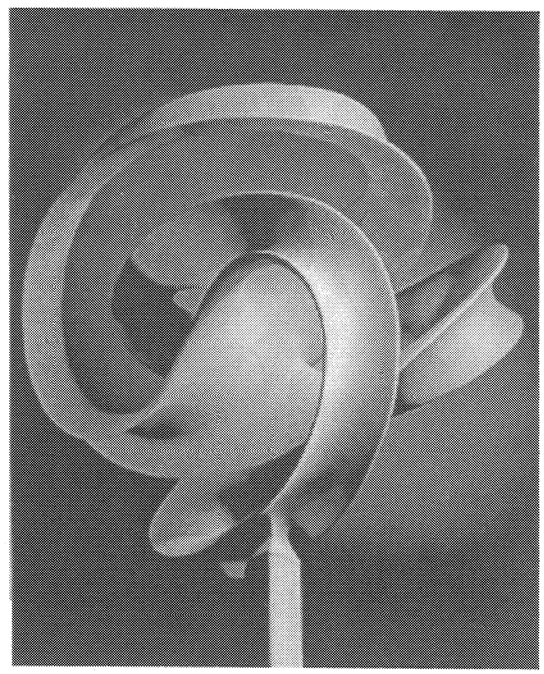

Figure 10: *"Atom", an "atomic flower" whose metaphoric electrons are negative curvature ribbons arcing from the surface of a sphere into its interior nucleus where they merge with the monkey saddle module from Scherk's third surface.*

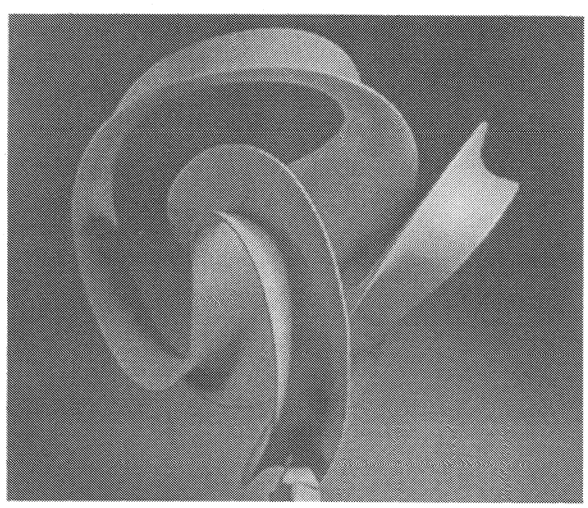

Figure 11: *Another view of "Atom", the first composition of the blended paradigm.*

Reformulating Gaussian Curvatures into a Ribbon: The Second Paradigm

In the succeeding paradigm, I essentially began deploying ribbons on the surfaces of spheres, tori, and eggs (Figures 7 through 9, respectively). In doing this, I oriented the ribbon's crescent cross-section to always turn away from the positive curvatures of these underlying geometries, except when modulated by a twist parameter keyed to the sectional curvature variations in the sinusoidal paths of the ribbons themselves. Through the calculus of this dual function system, I intuitively maximized the overall negative curvature of the ribbons in the hope of giving them an unprecedentedly fluid elegance of economy and coherence.

Holistically Integrating the Scherk Module with the Gaussian Ribbon: The New Paradigm

It is, then, the genomes of these two paradigms, aesthetic successes in themselves, which merge to form a third and perhaps more richly definitive aesthetic species (Figures 10 and 11). In this first formulation, three negative curvature ribbons arc around the surface of a sphere before diving into its interior at both their ends to seamlessly communicate with the negative curvature complex forming the sculpture's nucleus. This complex is actually a modular truncation from Scherk's third minimal surface. Metaphorically, the sculpture can be viewed as a kind of atomic flower whose electron arcs integrate with a denser nucleus. A trefoil dimension is present in the passage of some of the arcing ribbon's edges through the interior of the sculpture and out again. The sequel design now-in-progress also has an abstractable trefoil dimension, but is superior in that all the edges of its ribbons pass through the interior and out again opposite, as they would in an uncomplicated trefoil; rather than any of them remaining on the periphery as happens in the original composition. (In both cases, of course, these features are integral reflections of design.) In other words, the cogency of the trefoil aspect is more fully realized in the newer composition, while the hyperbolic nuclear complex is more ensconsed in the depths of the interior than it is in the original. What is more its nuclear complex, rather than merely being an elementarily straightforward Scherk saddle, is based on a hexagonal ring of saddles closer to the Costa. But even the first composition of the blended paradigm successfully combines the organic richness of a nuclear hyperbolic complex with the more aerial grace of the sine-ribbon arcs.

Collaboration with Carlo Séquin: Virtually Prototyping the Paradigms

Computer scientist Carlo Séquin and I began collaborating extensively on the toroidally warped Scherk surface paradigm soon after I finished the initial composition which set its precedent (Figure 3). I must confess that in the beginning I was somewhat naively skeptical, but after several months of intense work on his end recognizable graphics were emerging from the earliest generation of the computer prototyping program for the paradigm which he had made his project. At the time this had a dreamlike quality unacclimated as I was to dialogue after the years of working alone. To my wonderment, Carlo would ultimately systematize the paradigm with sliding parameters whose scope was no less potentially than the virtual generation of every possible formulation. And even as well as I understood the paradigm, most of the subsequent virtual creations were otherwise beyond my unaided imagination. Some, reflecting a collaborative process of aesthetic optimization, have become actual mahogany sculptures carved from computer-templated laminations based on stratified slices through their geometry (Figures 4 and 6). (A discussion in greater depth of Carlo's program and our collaboration can be found in our companion articles for Leonardo [1 and 2], as well as in his contribution to the *Proceedings* of the first Bridges Conference [3]). The graphics potential of Carlo's prototyping program for this paradigm is exemplified in Figures 12 and 13. Recently, he has also developed an augmented sculpture generator (SG2) to prototype formulations of the sine-ribbon paradigm such as the simulation in Figure 14. The newest blended paradigm has yet to be parameterized for virtual representation, however, and may never be since its subtle design complications may be only formidably programmable, and it may lack the germinal potential to generate the extended families of engaging formulations inherent in that of the foregoing paradigms, which seems the implication of my present understanding...but as this continues to grow the potential for a more numerous periodic table of atomic flowers than I now imagine may be revealed, enough conceivably that every great particle accelerator now in existence could have one uniquely its own.

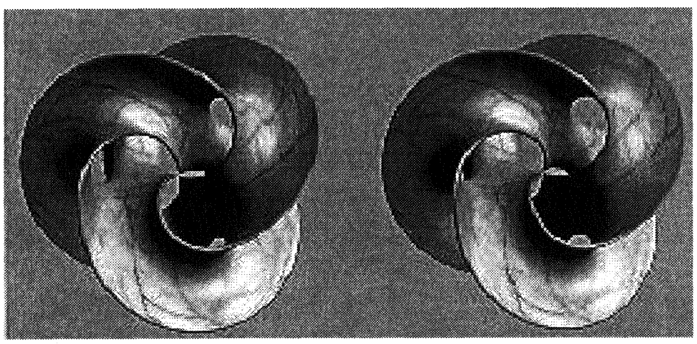

Figure 12: *A virtual stereo simulation of the mahogany sculpture seen in Figure 5 which differs from it slightly in the exact conformance of its edges to the surface of an underlying torus, so that rather than passing straight through interior, as those in the actual sculpture do, they pass over the curvature of the torus forming a trifoliolate edge pattern (Courtesy of Carlo Séquin).*

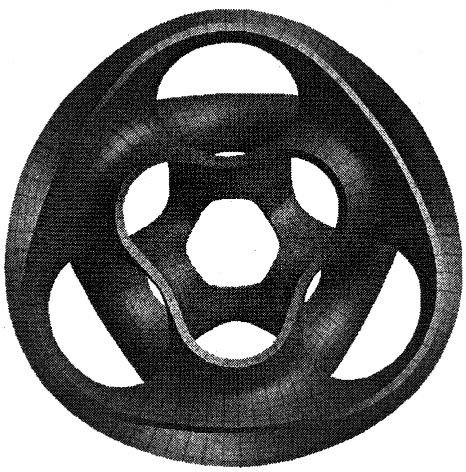

Figure 13: *A virtual simulation of the mahogany sculpture seen in Figure 4. These two are identical in that the simulation and computer-templates for the sculpture itself reflect the same parameterized geometry (Courtesy of Carlo Séquin).*

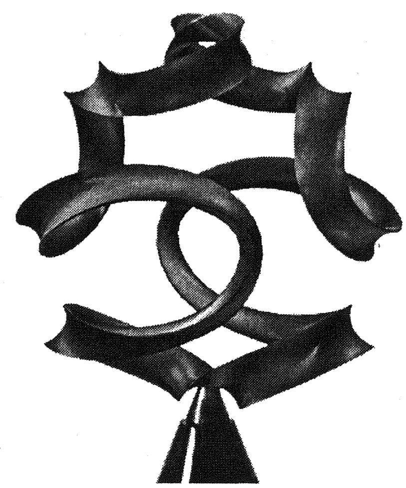

Figure 14: *Virtual simulation of the bronze ribbon sculpture seen in Figure 7 showing the augmented capabilities of the second generation of Carlo's prototyping program (Courtesy of Carlo Séquin).*

References

1. C.H. Séquin, *Virtual Prototyping of Scherk-Collins Saddle Rings,* Leonardo, 30, 2, pp 89-96, 1997.
2. A.B. Collins, *Evolving an Aesthetic of Surface Economy in Sculpture,* Leonardo, 30, 2, pp 85-88, 1997.
3. C.H. Séquin, *Art, Math and Computer: New Ways of Creating Pleasing Shapes,* Proceedings, Bridges: Mathematical Connections in Art, Music, and Science, R. Sarhangi (Ed.), pp 1-10, 1998.