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Mathematical Connections
in Art, Music, and Science

Teaching Mathematical Thinking through Origami

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Abstract

In this paper, we offer general and specific strategies for using activities based on origami to invoke mathematical thinking. Origami refers to the Japanese art of paper folding. Background is given on origami, including the development and critical aspects of the practice of origami in the United States. Next, we indicate how origami can relate to the current goals of K-12 mathematical education. General strategies are described that can be applied to any origami model. The basic strategy is for students to do the teaching. Then we describe in detail a variety of models. The examples include a variety of different computer-based techniques that middle and high school students can use to produce supporting materials for teaching.

Background

Origami is the art of paper folding. By a sequence of folds, a flat piece of paper is turned into a stylized animal, flower, box, or other recognizable object, generally 3-dimensional and often with moving parts or serving a utilitarian purpose. The final object is called a 'model'. Origami is associated with Japan, but it is practiced all over the world. The classical models include the water bomb, crane, and flapping bird. In recent times unit or modular origami, in which geometric constructions are built up from so-called modules, has become popular. Origami is both a craft and an art.

Origami as practiced in the United States and elsewhere, has developed a certain culture, largely influenced by Lillian Oppenheimer, Alice Gray, and Michael Shall who over time established Origami USA. In this culture, everyone is potentially a teacher as well as a student; a high value is placed on sharing. Similarly, care is taken to giving credit to creators, people who add variations to models, teachers, collectors, and people who write down directions and diagrams. It is to be noted that the recommended practice in origami circles goes beyond the letter of the law concerning intellectual property. Certain attributes of the nature of origami and this culture provide the potential for its use in teaching (and doing) mathematics.

- Creating an origami model involves following a procedure.
- The procedure involves the spatial manipulation of the paper.
- Different models can be related to one another by examining the modifications of general procedures.
- Teaching the creation of a model involves communication, either orally or through directions (text and diagrams) or perhaps computer based hypermedia.
- Teaching in origami, by nature and convention, is cooperative, applied, and student centered.

In order to take full advantage of the potential of origami for supporting teaching and learning of mathematics, it is critical to have a broad, modern view of the goals of mathematics instruction. We are not just talking about teaching students arithmetic or the terminology of geometry. Starting around 1989, reform activity in K-12 education began that culminated in the development of a new set of standards for mathematics. In these standards, the focus is on:

- promoting the study of open-ended problems
- promoting the application of mathematics to actual problems
- encouraging mathematical discourse: students "talking" math
- developing in students the habit and skills to use mathematics to describe the world
- increasing attention to topics such as geometry and spatial relations
- adopting instructional practices that are 'hands-on' and 'minds-on', involve peer teaching and collaboration, and span disciplines.

The mathematics reform movement is consistent with the general educational theory called constructivism. This theory views learning as not being a simple transmission of information from expert to student, but as the construction of individual frameworks by the student through the assimilation of new information with old. Most of these ideas are based on the lofty goal that teaching and learning is successful only if students retain concepts and skills and can apply what they have studied to new situations. To put this another way, teaching and learning must shift from concentration on the surface features of memorizing terms and rote-practice of procedures to invoking deep enough understanding of concepts to support students' applying, comparing, and adapting what they have. This change is most likely to happen if the learning process itself involves confronting problems and applying concepts.

This view of learning opens the way and even creates a demand for changes in curriculum. Origami is a prime candidate for satisfying some of this demand. It even creates artifacts to take home. The multi-cultural and multi-generational nature of origami--it is practiced all over the world and by people of all ages--is an additional attribute that makes it appealing for the classroom.

A significant goal of current reform efforts is not simply an improved acquisition of mathematics knowledge, but an improved epistemological view. Students would view mathematics as concepts and procedures that are continually evolving in a mathematics community. Origami is reflective of this, with new models being formed based on old ones, modelers learning from one another, and communication as a key part of the process. In fact, origami can be considered a subset of the overall mathematics community and discipline.

In this paper, we will describe a set of general strategies that can be used for enhancing K-12 mathematics education through the making of origami models and then apply them to a specific set of models, ranging from simple to intermediate in complexity.

Strategies for any model

Here are general techniques that can be applied to any model and fit very well in the culture of origami, in which every student is a potential teacher.

- The most basic use of origami is to have students teach models to other students. Various strategies exist for this. You can divide the class into groups and teach each group one model in a set of models of similar complexity. Their anchoring task is to teach the model to the people in the other group. They can work together, trying different approaches, and then agree on one presentation, or they can each find a partner in the other group. Notice: even if there is one 'designated teacher', he or she may be using ideas from many students in the final lesson. Of course, you must manage the process over time so that the same students are not doing all the talking. You can listen and give feedback to rehearsals or you can just let them do it. During rehearsals or after the 'real' lesson, discuss explicitly the uses of language. Point out that specialized language - 'jargon' - serves a definite purpose. Origami and mathematics each have their own jargon and are highly useful in describing folding. Your students may develop their own jargon. Having students develop their own terms and reconciling those with the traditional terms illustrates both the arbitrariness and usefulness of conventions. Discuss also the viewing angle of the audience and the use of gesture.

- Assign students the task learning new models by consulting the many books available (see the bibliography) and also a growing number of Web pages.
- As a natural follow-up to (oral) teaching, give students the task of preparing directions using their own writing and diagrams. Preparing directions at the level found in books is a challenge. However, it is possible to make acceptable diagrams using a variety of methods. Computer based systems can be used and this can serve as an opportunity to encourage students to refine and polish their work. Many people use specialized tools to produce drawings, but even the basic draw tools can be beneficial. Other options are to scan in hand drawn diagrams, scan actual models in development, or use a digital camera to produce images. These images can be marked up using a computer drawing program. Finally, students can produce hypermedia: text, images, animation, and sound linked together, with the navigation under the partial control of the user. Several different techniques for producing diagrams are included in this paper.
- Ask students to consider beforehand what will be the results of making a fold. Ask them to visualize it 'in their minds'. Encourage them to pose generalizations on the effects of folds. For example, folding an edge to a parallel edge divides an area in half.
- Encourage students to compare models to models and folds to folds. Certain models (or partial models) are called bases in origami. Similar folds can be applied to different models in a way analogous to functions or transformations applied to different objects in mathematics.
- Ask students to describe and keep track of symmetries in models as the folding proceeds.
- Encourage students to ask themselves why a particular model works. For example, in the models described below that begin with a rectangle that is not a square, the model would not work if you began with a square. Similarly, you can ask students how there can be movement in action models.
- Ask students to compute a specific measurement of the final model in terms of the original dimensions. Moving from linear to two and three dimensions, ask students to determine what regions of the paper end up as specific regions of the final model. When in the folding procedure does a model become (truly) three-dimensional?
- The following strategy or lesson for origami is not especially mathematical, but it is important: neatness truly does count. In contrast to many traditional academic topics, there is less risk of over emphasizing neatness to the detriment of critical thinking or of failing to distinguish between mistakes of accuracy and mistakes in fundamental concepts. The distinction between failings in surface features and failings in deep concepts is clear, even explicit, in origami. Furthermore, the importance of neatness is pragmatic enough that most students will learn this lesson on their own. A mathematical connection is to note when folds can be less than exact.

Origami activity motivates the explicit use of geometric terms. However, it is important to keep in mind that mathematical learning can be taking place even in the absence of mathematics terminology.

Examples

The following are meant to be examples of the range of applications of origami to the mathematics classroom. Included in the examples are a variety of means of creating instructions, including images of folds. We have chosen tools that are likely to be accessible in K-12 schools and we use them in a casual way. While the images, therefore, may not approach professional quality, we wish to emphasize the point that what *can* be done is worth it despite its shortcomings. If students are motivated to work more diligently and seek out high[er] level tools, that will be wonderful.

How to make a square. Since 8.5 by 11-inch paper can be more plentiful than square paper, it is appropriate to teach students how to make a square from a rectangular, non-square piece of paper. One way to explain this follows:

1. Fold the short side over against the long side. This marks the length on the long side that you want to keep. A square is a rectangle with equal sides and so this will produce a rectangle with all sides equal to the original short side.
2. Fold the extra part over. Unfold and fold in the other direction. This weakens the paper along the marked line.
3. Tear the extra part off.

Magazine cover box. This fold is a common first fold to be taught. The model serves the utilitarian purpose of being a container for other models. Magazine covers serve well here. The model requires a rectangle. This is, in fact, a good question to ask students.

The first three diagrams were made using Visual Basic. This was useful for making dotted lines and also exact measurements. The images were 'captured' into Paint Shop Pro. The folding in of the corners proved easier to do in Paint Shop Pro using the zoom feature and erasing lines by writing with the background color. The procedure is:

1. Fold the paper in half along the long dimension. Unfold. Fold the edges to the centerline. Unfold.

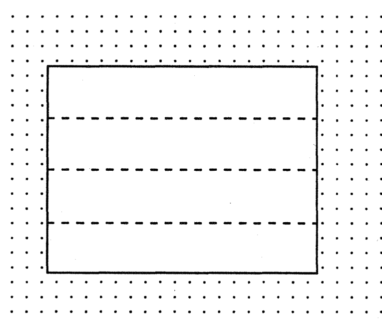

Figure 1: *Three valley folds in the paper*

2. Repeat in the other direction: fold in half, short side to short side. Unfold. Fold sides to centerline. You can ask about the symmetries in the model.

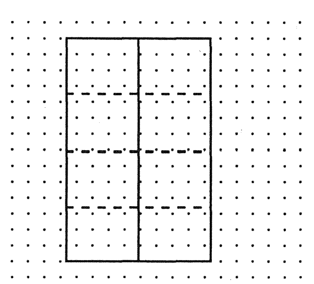

Figure 2: *The last two folds are called cupboard folds.*

3. There are three fold lines perpendicular to each folded edge. Fold a corner up to the nearest line.

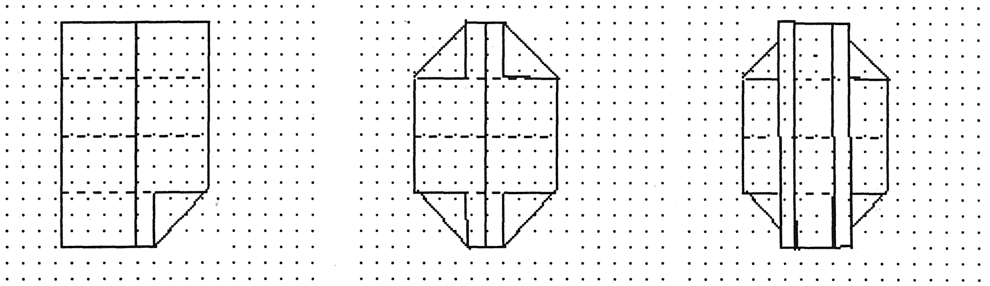

Figure 3: *First one and then all 4 corners folded, then flaps folded down over corners*

Why does this work? The answer is first of all, that the sides not being equal gives rise to the potential flap. The flap locks the corners. The locked corners mean the sides of the box stay together.

What are the final dimensions: sides of the base and the height of the box? Also, what is the size of the flaps? Consider framing this question in terms of the abstract L for long and S for short sides. This use of the algebraic paradigm arises quite naturally so take advantage! The following is a series of probing questions:

Let's go back in the folding procedure to when the short sides are folded into the center. You have two columns of 4 little rectangles. What are the dimensions of each rectangle? Is it similar to the original rectangle? The dimensions are ¼ of L by ¼ of S so, therefore, it is similar.

So now fold up the corners. The folded corner does not cover the whole rectangle. Why not? Because L is bigger than S. What is the linear measurement of the leftover amount? It is ¼ times (L – S).

Now we can calculate the dimensions of the model. The height is the side of the corners, which is ¼ of S. For the dimensions of the base of the box, look at it! The first folds divide the paper into 4 parts—What is a good term for these? You may get a spontaneous response that the long strips have thickness ¼ of S and length L and the ones going the other way, the short ones, have thickness ¼ of L and length S. The final box has a base two of the long ones by two of the short ones. So the base is ½ times S by ½ times L.

Now, measure the actual dimensions of the original paper. Substitute (or to use a more familiar phrase, stick in) the values. Then measure the model. Is it exactly the same? How exact? What are sources for any discrepancies?

Water bomb base. The water bomb is a traditional fold. The first few steps also form one of the fundamental bases used in a variety of folds. The diagrams below, showing (more or less) regular dashed lines for valleys and short and long dashes for mountains, were made using the common software program Paint. The folding procedure for the water bomb base is

1. Start with a square. Fold side to opposite side. (The origami term for this is a book fold. Any fold produces what is called a valley fold on one side and a mountain fold on the other. After doing some origami, the students will probably develop their own jargon to satisfy the obvious need.) Unfold. Repeat this with the other two sides, making a cross. Turn the paper over and now make two diagonal folds. (The origami jargon for this is a diaper fold. This could be told to the students for its historical significance.)

2. Place the paper on a surface so that the diagonal folds are mountain folds. The paper assumes the shape of a tent. (This is the same as the top part of the business card frog, so you can ask students if they have seen anything like this before.) Looking at the paper from above, you can form it into a 4-

pointed, star like figure. What can you say about the symmetry? Push the sides in more and let the paper collapse into a flat triangle. It is now bilaterally symmetric with two flaps on each side.

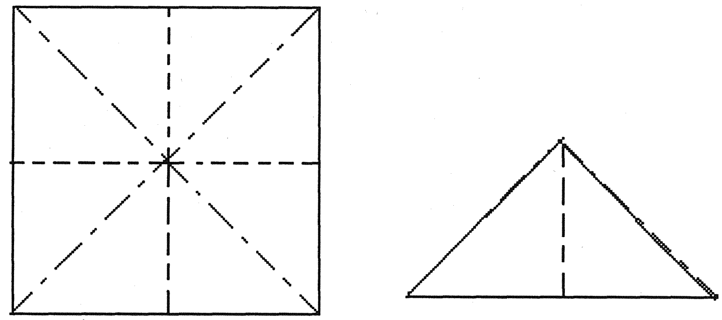

Figure 4: *Water bomb base: collapsing of initial valley and mountain folds*

Water bomb. To proceed with the water bomb model:

On each flap fold the point up to rest on the center point: four folds, two front and two back.

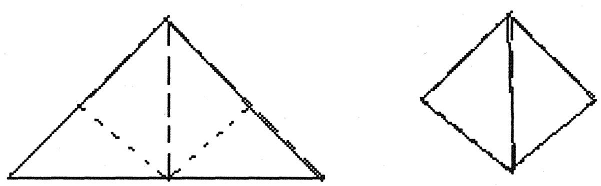

Figure 5: *Four flaps plus the center point at the top*

After the first fold is made. you can ask students to describe what is being done to the triangle. This step was repeated four times. You can ask the students after demonstrating each of the next several steps how many times they think each is repeated.

1. Fold in the side corners to the centerline. This is done four times, though you may also think of it as folding in the two corners to meet in the center and then doing that twice for the front and the back. This makes pockets. The diagram below is doubled in size (using the stretch option horizontally and vertically) from the previous images to improve readability.

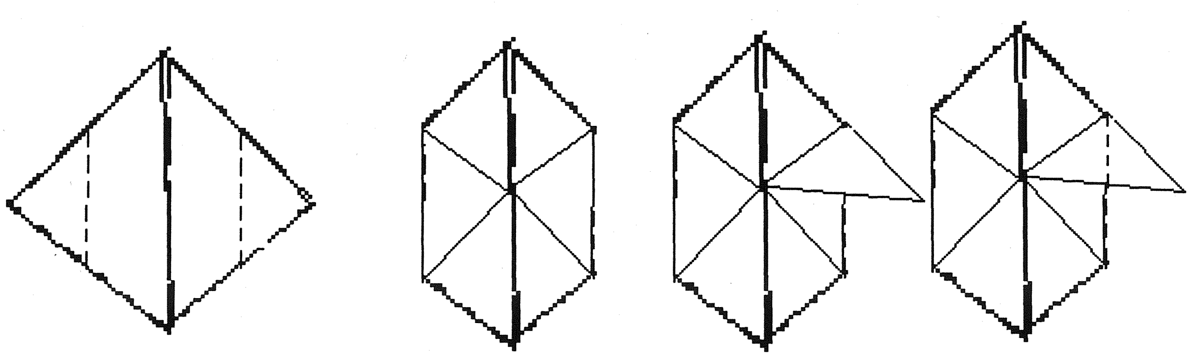

Figure 6: *Forming pockets and then preparing the tabs*

2. You will now do what is called locking the model by creating and inserting a tab. You make the tab by folding the loop flap out and then halving the triangle. Then you tuck the tab inside the pocket.

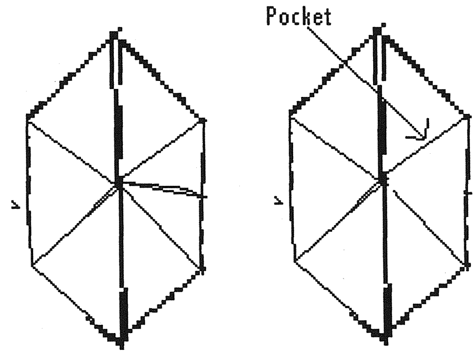

Figure 7: *Completing tab and then tucking it into the pocket*

3. Now to prepare for the next step in which the model assumes a 3-D shape and it is best to prepare for the shaping of the ends, fold the top and the bottom points to the center and unfold. Find the end of the model with an opening. Blow into the opening while pressing gently on the other end. The model will become a closed box.

Why does this work? You have created pockets and then put tabs into the pockets. This makes the model retain its shape. You can also think of the model as having a rough 3-D shape from the time you created to tent-like so-called water-bomb base. You just worked with it in a relatively flat form.

You can use the traditional water bomb, or any other model that assumes a three-dimensional shape, as a basis for questions on linear dimension, surface area, and volume. What is the volume of the cube that results from folding a water bomb from a square of a given size? What is the surface area of the cube? Each of these questions can be answered by a variety of methods. You can calculate the volume, but you can also measure the volume by filling up the water bomb with a known volume. Of course, metric measurements are helpful here. Similarly, you can calculate the surface area by a calculation based on the (calculated) dimensions of the cube.

To help calculate the surface area, you can color the surfaces of the water bomb and then unfold the model. The coloring does not have to be complete. Just make sure you mark every facet. It is also a good idea to mark the edges though these markings also need not be exact. Before unfolding, ask the students to guess (visualize/imagine) what (where) the colored areas will be. It is a separate and fun exercise just to add up these areas. A more elaborate exercise is to account for all of the original surface areas by keeping count of the areas of multiple thickness.

Re-folding the model, with the outside surface area marked, and following how that paper moves seems to reveal new sides, so to speak, of folding. I recommend this tactic for other models.

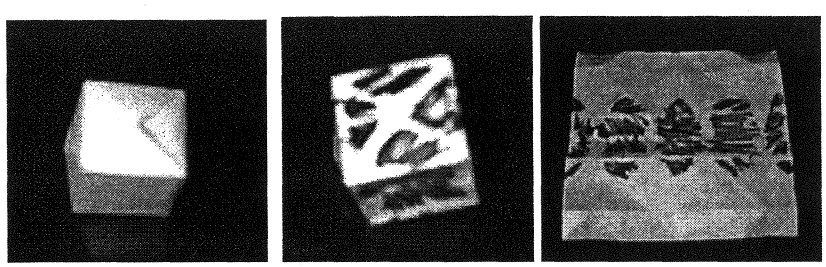

Figure 8: *Original water bomb; outside surface marked; unfolded (images produced using digital camera)*

Traditional frog. Since we are discussing two frog models in this paper, and, in fact, there are more, the reader and students may ask why? One explanation is that the word for frog is close to the word for 'return'. The wives of fishermen put frog models in the household shrines to indicate the hope for the safe return of their husbands.

Diagrams for the traditional frog are found in many books on origami. These were produced by placing the model on the scanner at stages in the folding. A cautionary note is that you must remember that the scanning is from below. The images were saved as Windows bitmaps, palette of 256 colors.

1. Make book (horizontal and vertical) folds on the white side of the paper and diaper (diagonal) folds on the green side.

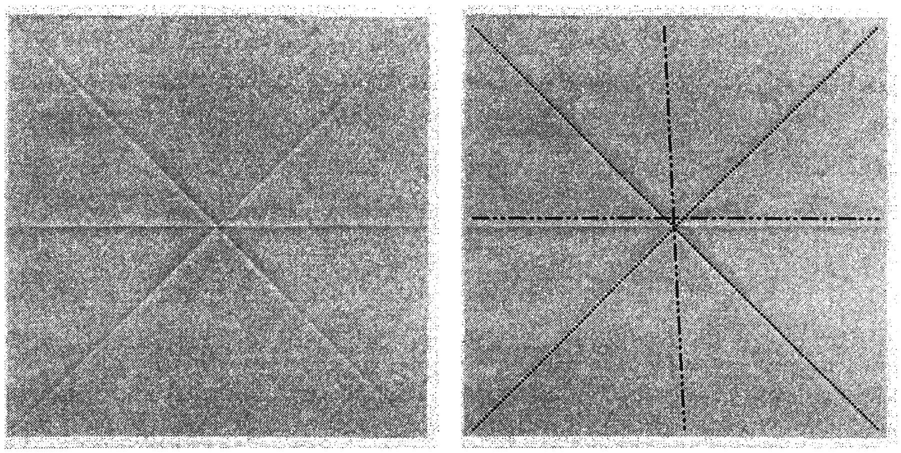

Figure 9: *Image from scanner and image with extra markings put on using Word draw facility.*

2. Collapse the model to form a diamond shape and then fold over flap to centerline.

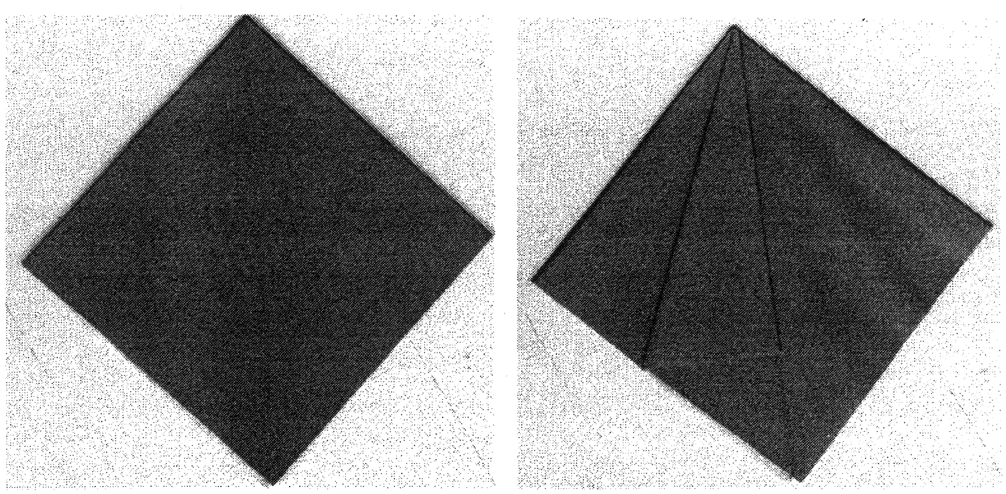

Figure 10: *Diamond actually one quarter of original size of paper.*

3. The squash fold is made by taking apart the fold just made, putting your finger in the flap, and then folding (squashing) it down. Repeat for each of the remaining 3 flaps.

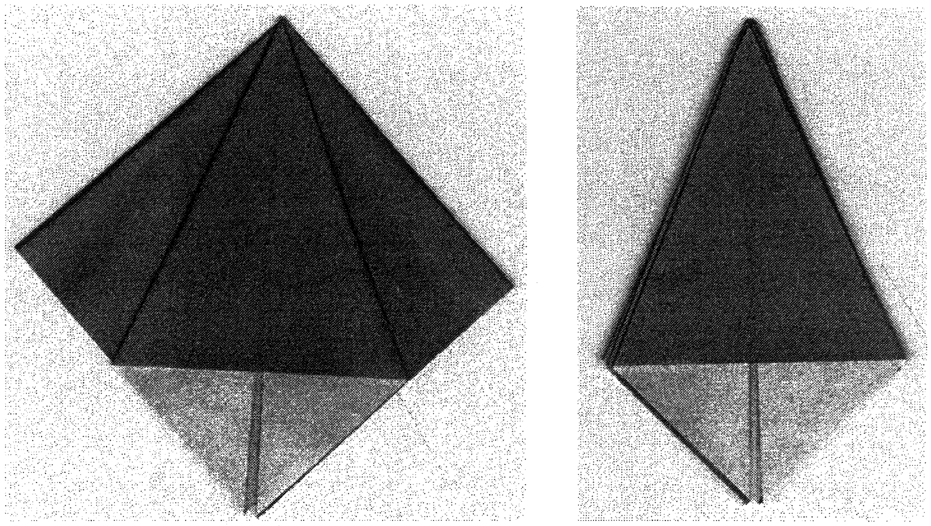

Figure 11: *Squash folds*

4. The next step is to make what is termed a petal fold: fold in the sides, unfold, and then pull up on the raw edge and let the sides collapse. The middle figure shows a so-called textbox and an arrow produced by the Word draw facility.

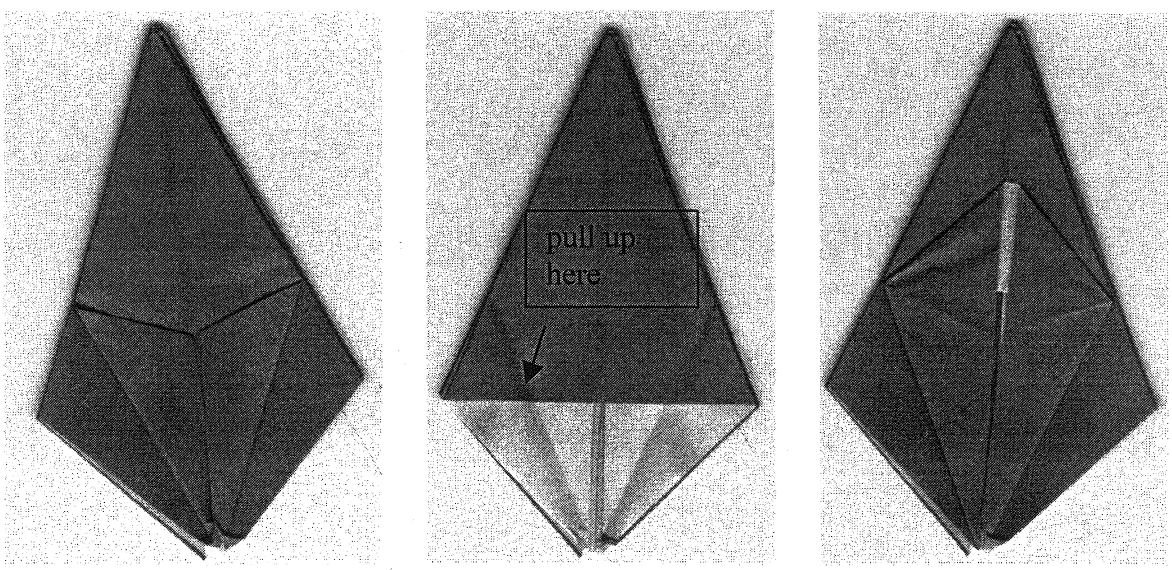

Figure 12: *Three steps of petal fold*

4. Repeat the petal fold three more times. The next folds are on the smooth sides of the flaps. Flip the flaps over like turning the pages of a book (a minor miracle in origami jargon). Repeat three more times.

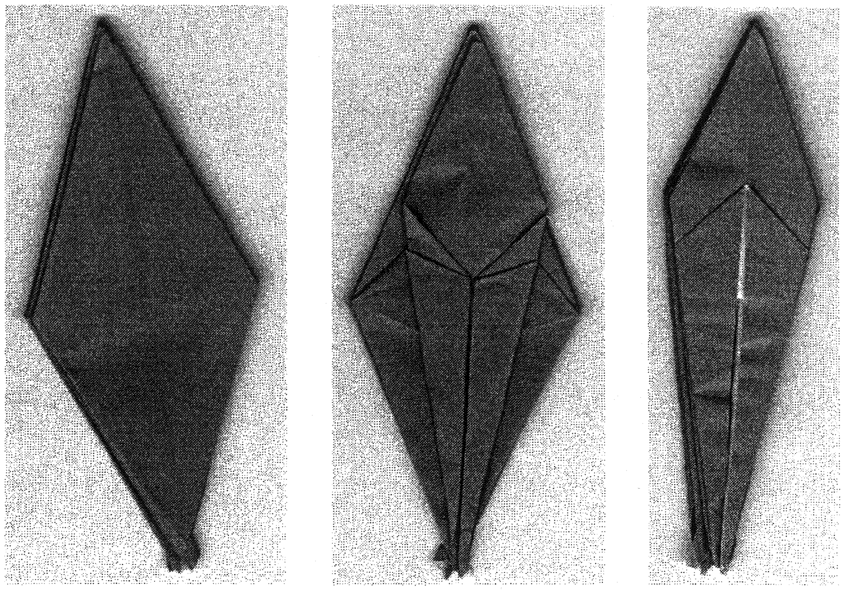

Figure 13: *Narrowing the body*

At this point, the model has four points that are the legs. The remaining folds shape the front and back legs, the front legs positioned towards the head and the back legs out to the sides. The fold used is a reverse fold (this is difficult to represent in diagrams, but fairly easy to demonstrate).

5. The back legs are positioned to point out to the sides. The image shows all four legs in place. Make joints in each of the limbs (think knees and ankles for the back legs and elbows and wrists for the front legs). Blow into the frog to make it fatter. This last image was done still using the scanner by propping it up so that it wouldn't crush the model.

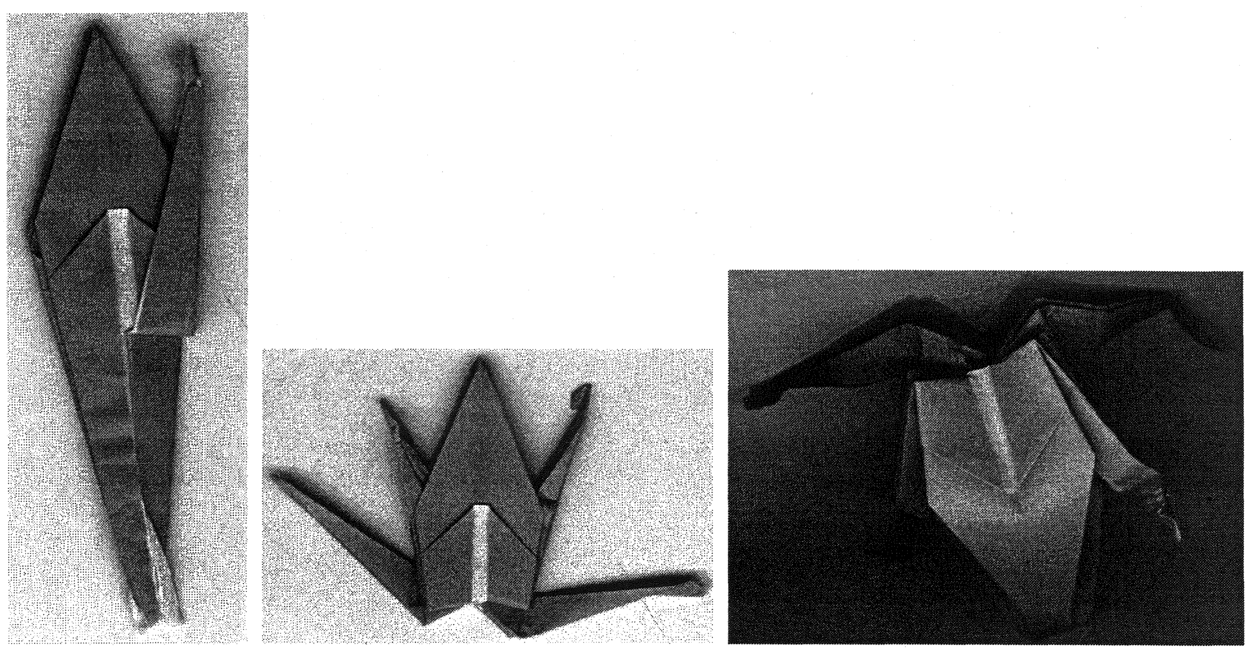

Figure 14: *Forming legs and then inflating the body*

The frog begins with what is called the preliminary base. It is made by the same folding pattern that produces the water bomb base: You can encourage students to examine this fold, figure out which folds are 'closed', and imagine themselves inside the model, looking out. It may be worthwhile to build a giant version and let the students stand or sit under it when it collapses around them.

The folding process for the frog reveals certain symmetries. For example, many folds are done in sets of four, starting with the four flaps of the preliminary base. What are these symmetries? When is it useful to consider the model flat, more-or-less two-dimensional, and when is it appropriate to consider it three-dimensional? Some directions indicate that you are to blow into the frog. How are the folds similar to the water bomb? How does this frog compare to the business card frog. Note: if you keep the frog flat (do not blow into it), it will jump in the same way as the business card frog jumps. How do the final dimensions of the frog compare to the size of the original square?

Dividing into thirds by estimation. Many folds offer the challenge of dividing the sides of a square into thirds. Students can be offered this challenge and then asked to compare methods. Two useful methods are the following:

What some origami folders call 'making an S' or simple trial-and-error consists of estimating a third from one side and then dividing the other side, that is, dividing the estimated two-thirds portion, and making a mark. You then fold again from the original side. The new mark generally will be different from the original and represents an improved estimate. One iteration is often sufficient, or you can keep going. The question is why does this work? That is, why do the estimates get better and better? This should be in the reach of students familiar with limits, though I frankly doubt that students who have just studied book procedures can transfer their knowledge to this domain. Instead, I think this exercise can and should be done to provide a basis for future work in limits. This is what educators call scaffolding.

Take a very long strip of paper, perhaps from a roll of wax paper or aluminum foil. Estimate one third of this length and mark the paper (call it A). Now fold from the other side, that is, divide the 'two-thirds' side in half. Make a mark (call it B). Fold from the original end to the new mark (call it A1). If you repeat this procedure, the marks A, A1, A2, etc. should get closer together and approach the real one-third mark. You can measure the edge, either before or after, to ascertain this.

What is happening is that if the original error, the deviation from the one-third point, was E, each fold halves the error. To explain this algebraically, if, starting from one end, the estimated (guessed) one-third point was L/3 + E, then the distance from the other end is L*2/3 -E. (I use the asterisk, *, for times.) Folding the long portion up to the guess is equivalent to dividing that length by 2: (L*2/3-E) * 1/2 is L/3-E/2. That is, this mark is E/2 from the correct position. As you continue doing this, back and forth, each fold improves the estimate by reducing the error by one-half.

Another way of demonstrating this is to take a very long strip of paper, measure the one-third point, and purposely choose a point distinctly different. (Perform the procedure and measure the differences.) It takes two folds, each one halving the error, for each iteration. So if A is E away from the actual point, B will be E/2 off from the desired (2/3) point, A1 will be E/4 away, A2 E/16 away, and so on. It is easy to see that the casual "making an S" method can quickly get to a satisfactory estimate.

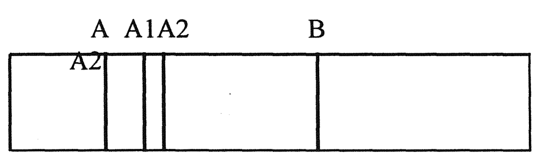

Figure 15: *Improvement of estimated point*

A side point for this is that my naming scheme of A and B and A1, A2, etc. is arbitrary. It does have some appealing aspects to it, but your students may devise schemes that appeal to them more. In any case, it is important to point out what is arbitrary and what is not. The symbolic or algebraic approach complements the experimental exploration. These different approaches to verifying that the iterative approach works provide different entry points to the topic for the students.

Dollar bill rosette. The dollar bill rosette model, originated by Paul Jackson and modified by Martin Kruskal, offers two compelling mathematical notions based on the requirement to mark off divisions of 11ths on the length of a dollar bill: one is the improvement of the estimate, and the other is a special property of the number 11. The latter is a property well known to mathematicians specializing in number theory. It offers evidence to students that there are properties other than even versus odd and prime versus composite. For the dollar bill rosette model, the property means that a certain procedure will 'hit' all the 10 intermediate points where you need a mark to divide the bill into eleven parts.

Take a dollar bill and estimate 1/11 on the long side by making a pinch. What you will be doing is using that estimate to mark off each of the ten sub-divisions and eventually return to the first one. The new mark will be an improvement on the original.

1. The first mark indicates 1/11; we will call it one part. If the short end is one part, then the longer portion is 10 parts. You have 10 parts and 1 part.
2. Fold the long portion in half (make a pinch) and now you have 5 parts and 6 parts.
3. Divide the 6 parts in half, and get 8 parts and 3 parts.
4. Divide the 8 parts in half, and get 4 parts and 7 parts.
5. Divide the 4 parts in half, and get 2 parts and 9 parts.
6. Divide the 2 parts in half, and get 1 part and 10 parts.
7. Repeat this process. You are working from the other side, and will eventually get back close to the original mark.

Think about what you have done and why you could do it. At each stage, you have the dollar divided into two pieces, one being an even number of parts (11ths) and one an odd number of parts. (Why is this true? Answer: eleven is an odd number so partitioning into two whole number parts always yields one even and one odd portion.) You can always divide the even portion into two pieces of the same size. Keep going until you return to the starting configuration.

What is true of 11 and certain other numbers is that this procedure goes through each of the numbers 1 to 10 before getting back to 10 again. It may be more fitting to say that it goes through the sequence (10,1) (5,6) (8,3) (4,7) (2,9) (1,10) (6,5) (3,8) (7,4) (9,2) (10,1) instead of reaching (10,1) earlier. You may consider letting your students develop their own notation to express a mark that divides the bill into portions representing A and B number of parts. Try other numbers! You can also ask students to examine how other numbers satisfy the physical and aesthetic requirements for the model.

Leaving number theory, we can use the same principles as the situation with thirds to prove that if the original error was E, the error when you get back to the first mark again is E/2^10 (2 raised to the tenth power) which is E/1024. This is because you have made 10 folds and each fold halved the error.

Here are diagrams for the rest of folding made by scanning in hand-drawings along with an actual, final model.

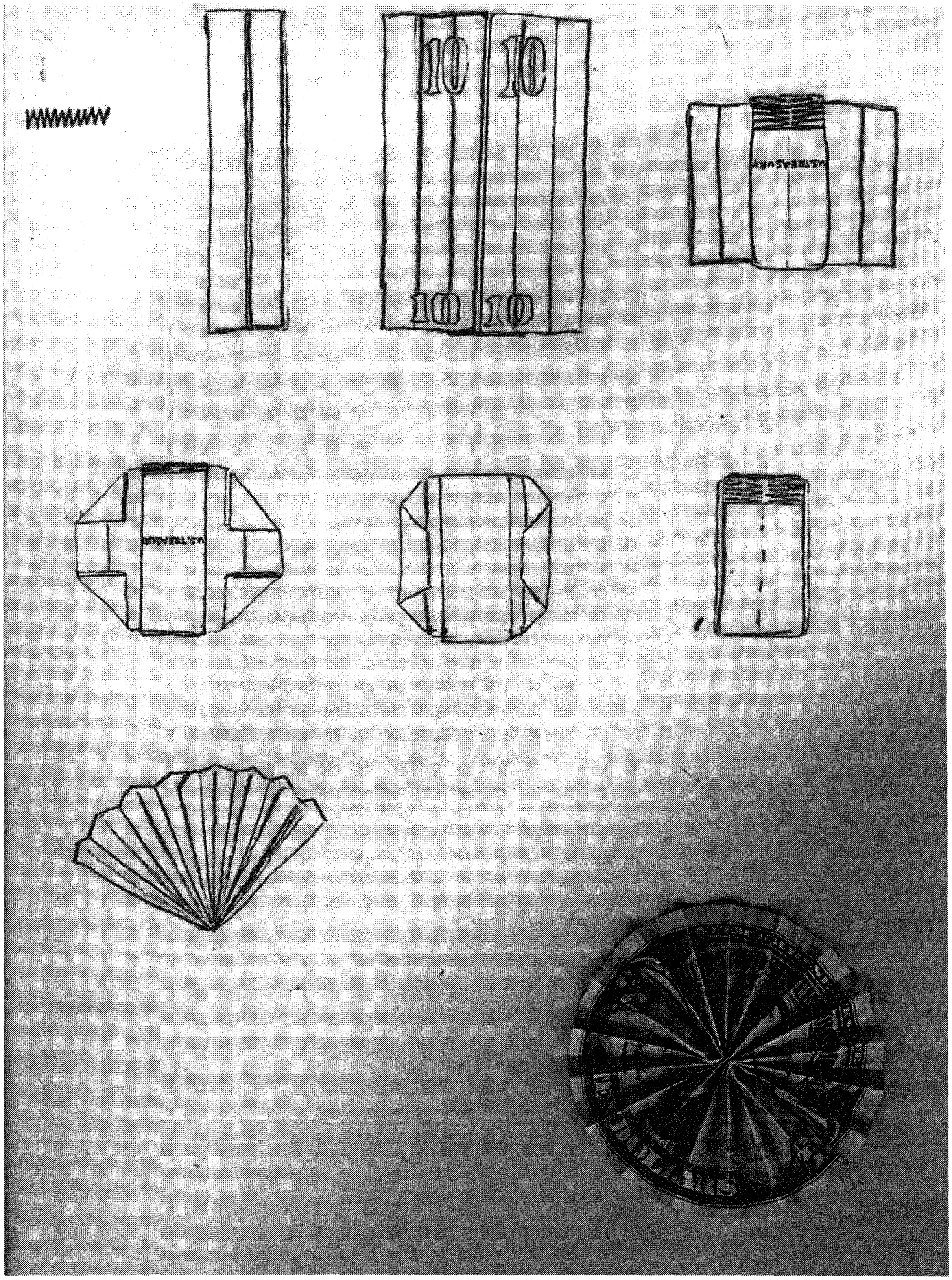

Figure 16: *Made by scanning drawings with actual model*

1. After marking off 11ths (begin again with the improved first pinch and go through the sequence folding valley folds each time), make an accordion fold by dividing each of these folds in half. That

is, you will have a sequence of pleats with 22 panels. The fold dividing the 11^{th} is in the opposite direction so you have alternating mountain and valley folds.

2. Arrange the pleats so that they are divided in half. Then unfold three panels.
3. Fold the model in half.
4. You now want to attach the two sides. Make tabs to lock the model by folding down the corners, rolling the edge and tucking the modified flaps into the model.
5. Pop the model open.

The tasks and questions described for the traditional models can also be used with the dollar bill rosette. For example, students can be asked to determine the radius of the circle. Why 11 (or 22?)? Students can try alternatives to see how they work, perhaps 8 (16) or 16 (32). You can also assign the task of teaching the model and explaining each or any step of the procedure. Students may want to race through the moves, but make clear that the sign of a well-done job is that members of the audience master the model.

Conclusion

Origami does not require fancy equipment. Ordinary paper, even used paper, works well, especially for the early stages of learning. However, when students become teachers, it can be advantageous to make use of computer technology to make diagrams and other teaching aids. Several methods were demonstrated in this paper. Such computer use often serves to make the activities even more appealing to students and to raise the standard for student work.

Origami can be an answer to the demand from educators and from others for activities and entrée points to mathematical discourse and applications called for by the National Council of Teachers of Mathematics and other reform efforts. One of the most frequently stated goals of such educational reform efforts is to change the role of teacher from lecturer to guide, from the teacher being the only source of knowledge in the classroom to the condition of the classroom being a community in which everyone is an enthusiastic, responsible, and contributing teacher and learner. In the origami community, we have long appreciated the pleasure and value of being teachers and learners together and we can bring these experiences to the classroom. Origami as an activity, with its open-ended nature, communication, and interconnectiveness, is reflective of the epistemology of mathematics that we want our students to experience.

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