Bridges
Mathematical Connections
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The Well-Rounded Architect: Spheres and Circles in Architectural Design

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Abstract

It is often stated that ancient architectural monuments such as the Pantheon were constructed as models of the Cosmos. An integral part of that interpretation is the notion that the architectural form of the dome represents the heavens. Ancient cultures created two kinds of models for the universe: analogue models represent their objects through physical resemblance; digital models represent their objects as functions of time. While the idea of roundness appears to be one of the primordial distinctions made with regards to the heavens, round buildings present particular problems that the architect has to solve before he is able to construct a round building as a cosmic model. This paper traces ideas of roundness and the notion of heavenly events as a function of time in ancient culture. It examines the nature of circular and spherical forms as structure. Finally, this information is applied to an analysis of the Pantheon in order to reveal its triumph over of structural problems as well as its function as both an analogue and a digital model.

1. Introduction

From the dawn of time to our present day, the efforts of man to understand the world in which he lives and his efforts to create his own built environment have gone hand in hand. No greater evidence of this is found than in sacred architecture, because building a temple to a deity gives formal expression to religious beliefs, transcending the reality of cold stone and marble and making evident to all men what was divined by a privileged few. As an architect, I am fascinated by the use of circular and spherical geometry in architecture: round buildings are hard to build and require great technology and wealth, so there has to be a strong motivation to build them. Religion provides man with perhaps his strongest motivation; probably for this reason we have a tendency to interpret archaeological remains such as Stonehenge as the remains of structures constructed for religious reasons. (Perhaps we tend to think that only religious reasons could provide motivation enough to move stones weighing 35 tons and above.)

While most cultures have devised some kind of model to explain their impressions about the universe, not all cultures were concerned with geometric models. The Egyptians and Babylonians had no such models; the Greeks, Romans and Renaissance Europeans did. It appears that in epochs in which geometric models of the universe involved spheres, we find some round sacred buildings, and where they were not, sacred buildings were shapes other than round. A survey of architectural history reinforces this theory, because we find concentrations of round buildings mainly in the Roman and Renaissance eras; in the Middle Ages, when the earth was thought to be flat and even rectangular, as hypothesized by Cosmas of Alexandria, sacred buildings were mainly rectangular. With a nod in the directions of philosophy and science, histories of architecture provide a summary account of the significance of round buildings: they were models for the cosmos. My intention here is to examine how this is so. To do this, I want to show how answers to questions about the cosmos might have found their way into the architecture of what is arguably the world's finest round building, the Pantheon.

This is a truly interdisciplinary study, involving architecture, geometry, astronomy and theology. When hunting and gathering societies gave way to agricultural and trading societies, knowledge of time, seasons and navigation provided a practical stimulus to the study of the heavens. But that the stars were also deities is shown by the formulation of an ancient Babylonian mathematical problem, "How much is one god beyond another god?" [1] How could the stars, the sun and the moon have been other than gods? Their movements in the heavens controlled the floods, the seasons, the planting, the harvest. Astronomy was therefore at the service of religion, and at the service of astronomy was mathematics, for it was through mathematics that celestial events were expressed and predicted. To see how architecture relates to astronomy and mathematics, it is necessary to examine the nature of models.

2. The Nature of Models

As mathematicians well know, there are models and models. A comparison of two of the most ancient models will illustrate the difference. The ancient monument of Stonehenge, built between 1900 and 1600 BC, is usually interpreted as a religious structure, but it has recently been argued that it may not have been a "temple" at all, at least not in the usual religious sense, but rather a massive computer (like a mainframe) for predicting astronomical events such as lunar eclipses [2]. Using the massive upright stones, events were predicted by means of a physical analogue ("when the moon is between these two pillars, the solstice occurs"). In comparison with Stonehenge, the clay cuneiform tablets of the ancient Babylonians were "pocket calculators". The Babylonians (4000-1250 BC) used arithmetical and algebraic models to represent the movements of the stars and thus predict the same significant occurrences by means of an abstraction, through calculation (the lunar cycle is x days long, the solar cycle is y days long, therefore in z days the solstice occurs). Though both models were relatively accurate, the analogue model involved a geometric arrangement of objects, while the digital model possesses no formal qualities [3]. Stonehenge's physical resemblance to the object it models, however, makes it suitable as a symbol, a possibility lacking in the Babylonian model.

Architects of sacred structures would naturally be attracted to a geometrical model of the cosmos because of architecture's affinity to geometry. Architecture almost always answers to a geometrical description, its shapes are usually defined by straight lines and planes, or by arcs and circumferences [4]. This is due in part to the tools that architects use in developing their plans and that laborers use in construction. In the past, triangles, also known as "set squares", were among the used tools of the architect; such geometric tools lead almost automatically to the geometricization of form. Modern computer-aided design systems have tended to adapt the existing techniques to a new technology.

Sacred architecture is no exception in being characterized by geometry, but it is a special case. Houses, factories and schools are all composed of geometric shapes, but we generally interpret the shapes as deriving from the building's function and not as symbolically meaningful in themselves. On the other hand, part of the sacred building's function is to be symbolically meaningful, so if shape plays an important role in a culture's cosmology, it is likely to play an important role in its architecture. Thus in round buildings we are mainly concerned with architecture as an analogue.

Having made a distinction between kinds of models, however, it is important to make a further distinction about what kind of a model architecture can be: buildings, however inspired by discoveries in cosmology, aren't really models of the cosmos; rather they are models of a model of the cosmos. Cosmic symbols in architecture are the result of a double translation, where ideas about the nature of the spherical nature of the universe are first translated into geometry, then retranslated into architecture. The translations are not literal; it is obvious, to paraphrase Robin Evans, that churches are not intended as precise models of the cosmos, for if they were, they would look like armillary spheres [5]. Spherical forms in any case represent a kinetic problem in architecture, because human beings move most easily on a horizontal surface.

Aspects other than geometry figured into the understanding of the cosmos: the nature of circle and sphere; the measure of *pi*; the relative sizes of the heavenly bodies. So we will find architecture is a synthesis of ideas related to geometrical physical resemblance and ideas expressed through other kinds of mathematics.

3. The Shape of the Universe

Probably the clearest impression we have when we look at the sky, whether on a clear day or on a starry night, is of its roundness. Every culture has recognized and expressed the forms of circle and sphere [6], but not all of them have speculated on the shape of the universe. The Babylonians concentrated their efforts to understand the universe on tracking the movements of the stars rather than inquiring into questions of form; they developed a science of astrology, but no geometric model. The Egyptians were the first to differentiate the planets (the "wandering stars" from the Greek *planar*, to wander) from the fixed stars, but neither did they seek a geometric explanation of the heavens. The Greeks were the first to seek to understand shape. In the seventh century BC, Anaximander, through his observation of the apparent path of the sun's orbit, or ecliptic, proved that the earth is a sphere, relating the roundness of the sphere against which the stars were fixed to the roundness of the earth. The idea of uniform roundness was transferred to the movement of the planets in the earliest Greek model of the universe, developed by Philolaus in the fifth century. Philolaus was a disciple of Pythagoras and, like his master, believed in the supremacy of numbers, especially the perfect number or *tektraktis*, 10. His hybrid cosmic model, part numeric and part geometric, was based on the assumption of a perfect number of planets; because the only known planets [7] totaled nine, Philolaus had to invent a tenth planet, the "counter-earth". In his system, the planets revolved in regular circular orbits about a central fire. He overcame the problem of why neither the central fire nor the counter-earth were visible by explaining that, since the earth was immobile, its inhabited face was always turned away from the central fire and counter earth.

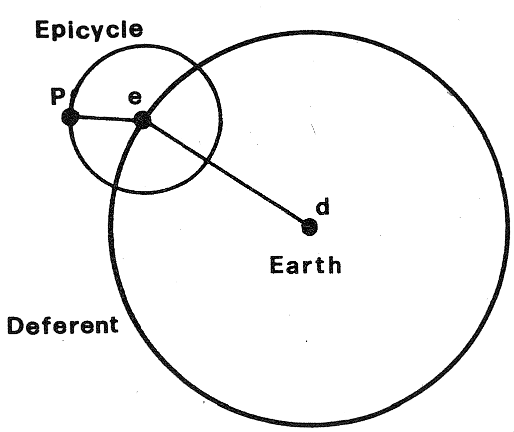

Figure 1. *Apollonius'* ***epicycles****. The apparently irregular motion of the planet P is explained through the exclusive use of circles by having it orbit about an epicycle with center "e", which orbits around a deferent circle with center "d" (representing the Earth).*

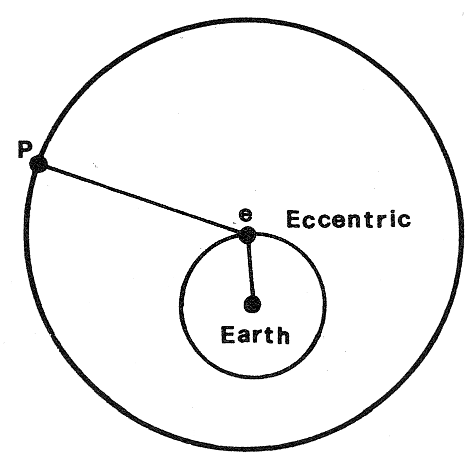

Figure 2. *Apollonius'* ***eccentrics****. Planet P's orbit is defined by a circle with center "e", which itself orbits in a circle about the Earth.*

A century later, Plato turned his attention to the structure of the universe. Because the circle and sphere, by dint of their uniformity and infinite symmetry, are perfect forms, Plato maintained the sphericity of the universe [8], and posited the sphere and circle as archetypes for the creation of other forms, such as the human head [9]. Plato posed the problem of finding a geometric model that would "save the phenomenon" [10], asking, in other words, for a model that would explain in a consistent way apparently contradictory phenomena. The problem was addressed by Eudoxus of Cnidus, whose system of uniformly rotating concentric or "homocentric" spheres revolved about a fixed earth at the system's center [11]. The plausibility of Eudoxus' model allowed Plato to retract what he had said about the inconsistency of movement of the stars [12] and to replace it with a statement that "the very reverse is the truth" [13].

Eudoxus' model was acceptable as a general representation of planetary motion, but lacked precision. Apollonius of Perga devised two alternate systems to enable more accurate predictions of celestial events, while still accounting in a regular way for apparently irregular movement. The first system made use of *epicycles*; the second *eccentricity* (Figs. 1 and 2). An epicycle is the small circular orbit of a planet, the center of which lies on the circumference of a larger circle, called the deferent; the earth is at the center of the deferent. In the eccentric scheme the planet moves along the circumference of a larger circle, the center of which travels along the circumference of a smaller circle, at the center of which is the earth.

Apollonius' model was further refined by Hipparchus of Rhodes in the third century BC before reaching its perfection in the hands of Ptolemy of Alexandria in the second century AD. Although Ptolemy lived and worked after the building of the Pantheon, I want to introduce his notion of the *equant*, yet another geometrical construction devised to replicate the motion of the planets through the exclusive use of circular forms, because it illustrates the great lengths to which astronomers were willing to go to preserve the notion of uniform roundness. Ptolemy's model presumed that the motion of the planets appeared irregular because we were looking at them from the wrong point of view. His corrected version of earlier cosmic models assumes that the planets moved in a circular orbit along an epicycle, which in turned orbited about a deferent that, however, was not centered on the earth but instead on an independent center. On one side of that center was the earth; equidistant on the other side was the equant, the point from which the planetary motions revealed their regularity (Fig. 3). Ptolemy's system of "wheels within wheels" remained *the* cosmic system for almost 1500 years, until Copernicus' time.

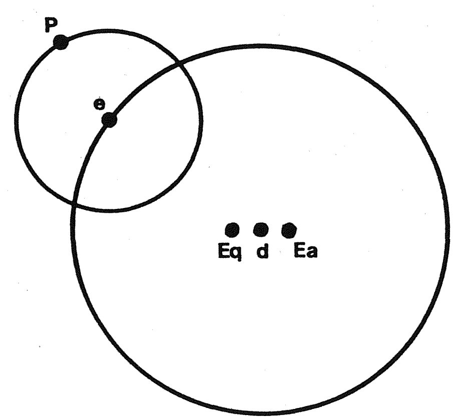

Figure 3. *Ptolemy's **equant**. Planet P's orbit is defined by a circle with center "e", which itself orbits about a deferent circle with center "d". The Earth "Ea" and the Equant "Eq" are equally spaced on either side of "d"; planet P's motion, irregular when viewed from the Earth, appears regular when viewed from the Equant.*

4. Construction the Model: The Architect's Knowledge of Circle and Sphere

The philosophical knowledge of circle and sphere as perfect forms for the universe is one thing; the technology for constructing buildings with circular and spherical forms is another. In this section I would like to show how technical difficulties were gradually overcome.

The Egyptians possessed knowledge about the circle that would be of use to an architect: their calculations of circular areas would have allowed the architect to order the marble for the pavement, and their calculations of circumferences would have allowed him to know how many bricks had to be made in order to build the walls. It is believed that part of the information transmitted in the Rhind Papyrus, one of the main sources of our knowledge about Egyptian mathematics, may have been passed down from Imhotep, architect to Pharoah Zoser; if so, we have the first direct connection between architecture and mathematics! [14] Key to calculating circular areas is understanding the nature of the ratio between the diameter and the circumference of a circle, the constant we now call *pi*. In Problem 48, the area of an octagon is derived from the area of the square in which it is inscribed: the octagon is formed by trisecting the sides of a square measuring 9 units to a side, then subtracting the 4 corner isosceles triangles. Since the area of each corner square is 9 (3x3), the area of each of the 4 isosceles triangles is 9/2; the area of the octagon is then 9^2-4(9/2) = 63. It must have been assumed that the area of the octagon did not differ substantially from that of the area of a circle inscribed in the same square, 63 being justifiably close enough to 64, the area of a square measuring 8 units to a side. Problem 50 describes the area of a circle measuring nine units in a diameter as equal to the area of a square with a side measuring eight units. If we substitute these values in the modern formula,

$$A=pi(r^2),$$

we find that 64 = $pi(9/2)^2$ and therefore *pi* is equal to $4(8/9)^2$ or 3-1/6, an error from the actual value of only .6% [15]. This value for *pi*, $4(8/9)^2$, was used again in the rule for finding the circumference of the circle: the ratio of the area of a circle to its circumference was held to be equal to the ratio of the circumscribed square to its perimeter.

The Babylonians also approximated a constant value for the ratio between the diameter of a circle and its circumference. Clay cuneiform tablets found in Susa in 1936 showed calculations comparing the areas and squares of the sides of triangles, squares, pentagons, hexagon and heptagons, as well as a comparing the perimeter of the hexagon to the circumference of the circumscribed circle. The ratio given indicates a value of *pi* as 3-1/8.

Archimedes of Syracuse refined the calculations of *pi* by extrapolating the method of the Babylonians to the comparison of the circumference of a circle to a 96-sided polygon, producing upper and lower limits for the value for *pi*,

3-10/71 greater than *pi* greater than 3-10/70.

This result, published in *On the Measurement of the Circle*, is certainly accurate enough for the purposes of the architect, and wasn't surpassed until the invention of the calculus.

Archimedes also made great discoveries with regards to the volume of the sphere, publishing his results in *On the Sphere and Cylinder*. His discovery that the ratio of the volumes of a sphere and the right circular cylinder of the same diameter in which it is inscribed is 2:3 was so gratifying to him that he had the figures carved on his tombstone.

Since the Greeks possessed the knowledge of circular areas and circumferences that would enable the architect to do accurate estimates of quantities of materials and they had a philosophical justification for the use of such plans, so it is somewhat surprising to find only a limited use of circular buildings [16]. The circular buildings that exist, the *tholos* (the Greek term for any round building) at Epidaurus, the Philippeion at Olympia and the Choragic Monument of Lysicrates, Athens are known only in plan; how they were actually roofed remains a mystery. The regularization of the theater form into an arrangement of concentric rows of seats introduced the circle into the Greek architectural vocabulary but begs the technical question of closure and, of course, is secular and not sacred. What was missing of course was the understanding how circles can be applied vertically in structures, that is, when the arch was understood.

In addition to the strength of the materials used in its construction, the great strength of the arch is due to its shape. This principle can be tested by trying to span the distance between two books with a strip of paper. If laid flat, the thin paper will collapse between the books; if placed between the books in the shape of an arch, it will not only support its own weight, but perhaps even the weight of another piece of paper [17]. The Roman arch was built of individual wedge-shaped stones called *voussiers* and a central trapezoid-shaped stone called the *keystone*, its name derived from the fact that when it is set in place, the arch supports itself. Neither half of the arch is self-supporting; for this reason Leonardo da Vinci defined it as "two weaknesses which, leaning on each other, become a strength" [18]. An arch requires bracing at its lower extremities, because the loads pressing vertically from the top tend to cause the lower ends to splay outwards. This requirement for constant bracing is probably why the arch is sometimes called "the structure that never sleeps" [19]. To construct an arch, a wooden support called a *centering* is built. Voussiers are laid from the bottom up on both sides until the keystone is finally placed at the top; at that point the centering is removed.

The structure of a dome is based on principles similar to those of the arch. The dome's infinite symmetry bring to mind the translation of an infinite number of arches about a center point [20]. The arch-like elements of a dome are called its *meridians*; horizontal rings circling the meridians are called *parallels*. The great hemispherical dome of the Pantheon in Rome is a good visual example of meridians and parallels, the interstices between them articulated as coffers. I stress that the Pantheon is a *visual* example because structurally it is built as a homologous shell of brick and mortar rather than constructed of meridians and parallels, and the coffers serve to lessen the overall weight of the structure (as well, of course, as ornamentation). This kind of shell structure is self-supporting because the bricks in the wedge-like horizontal courses are self-buttressing and cannot therefore collapse into the center; this is what makes the open oculus at the top possible. Domes, however, like arches, require buttressing at the bottom to resist their outward thrust. The Romans solved this problem by supporting the dome on the thick (20 feet!) cylindrical walls of the rotunda.

5. The Pantheon as an Analogue Model of the Cosmos

The Pantheon has awed generations of observers because of its vast scale and monumental simplicity (Fig. 4). No documents about the building, its design, or even the name of the architect survive. Dion Cassius, author of a history of Rome, wrote that the name Pantheon derives either from the fact that it was dedicated to the seven major deities or because it resembled the canopy of heaven [21] I have arranged my observations of the formal aspects of the Pantheon as a walk through the different stages in the development of cosmology that the architecture might be intended to model [22].

The overwhelming formal quality of the architecture is that of roundness. The elemental beauty of the right circular cylinder and sphere seem to have been as fascinating for the Pantheon's architect as they were for Archimedes. Archimedes calculated the ratios of the volumes of a sphere and the right circular cylinder of the same diameter in which it is inscribed; the Pantheon is composed of one-half the sphere and one-half the cylinder. The precision with which these forms are constructed (the Pantheon, in comparison to other contemporary Roman buildings, has been called "a miracle of precision" [23]) makes it clear that the architect was not content with expressing any kind of roundness, but insisted on *perfect* roundness. Thus the first characteristic of the cosmos modeled by the Pantheon is the perfect roundness of the sphere of heaven: we are now at the age of the Babylonians and Egyptians, who observed that the stars seems fixed against a sphere. That the dome was intended as the heavenly sphere was made clear by its articulation. Each of the coffers was probably originally decorated with a bronze star at its center; these would represent the stars of heaven. The oculus represented the sun [24].

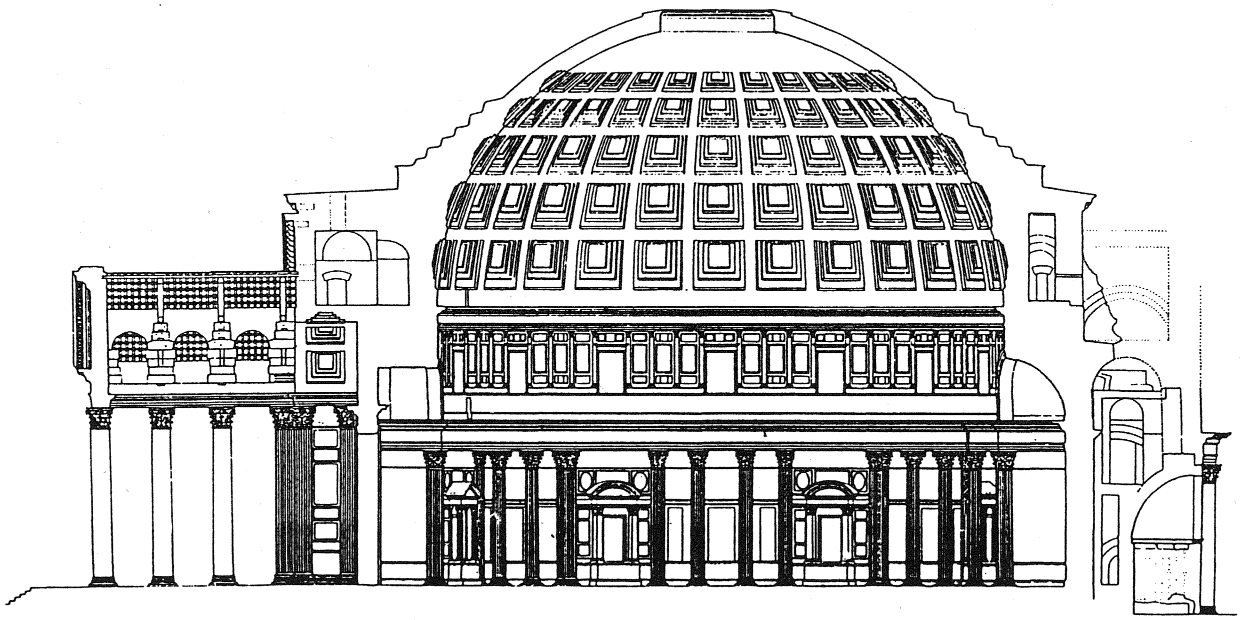

Figure 4. *A section through the interior of the Pantheon.*

Let us take the idea of perfect roundness up to the next stage of development, to the observations of Anaximander, who proved the sphericity of the earth. Now we notice that the floor surface is not flat but convex, being some 28 cm. higher in the center than at the perimeter. This suggests a spherical earth inside a spherical universe: uniform roundness. Three and a half centuries after Anaximander's confirmation of the sphericity of the earth, Eratosthenes of Cyrene calculated the circumference of the earth. He did this by using a scaphe, a sundial in the shape of a hollow sphere that had been developed by Aristarchus of Samos, to measure the angles of the sun. When Archimedes refined the method for computing the value of *pi*, it was possible to calculate the earth's radius. This knowledge would have made it possible to construct the Pantheon as a scale model of the Roman empire, as Pantheon scholar Gert Sperling believes. The diameter of the spherical dome was based upon a calculation of the area of the earth's surface occupied by the Roman Empire; the convexity of the pavement was intended to represent the curvature of that segment of the sphere of the earth [25].

The next level of observation takes us up to the observations of Eudoxus, who theorized a geocentric universe. The convex floor surface of the Pantheon represents the earth: the eight niches in the perimeter walls , one of which was the entrance, and seven of which may have contained statues of the seven deities associated with the planets, reinforce the ideas of a geocentric universe. The niches bring us up to the contrivances of Apollonius in their resemblance to epicycles, though their shape alternates between round and rectangular.

All of these formal characteristics contribute to the function of the architecture as a model for the universe. There is at least one major problem with the model, however. The proportions of the forms are such that height of the cylindrical walls is equal to the radius of the hemispherical dome, thus the sphere of the dome could theoretically complete itself, arriving at a point tangent to the pavement. Thus the earth and the cosmic sphere are represented as non-concentric. I made the point earlier that architecture is not a literal translation of a cosmic model, and I wish to add that , thus one could suppose that the eccentricity of the two round forms represents a recognition of the impossibility of constructing an exact model. It appears, however, that the architect gave this problem special consideration because he derived an ingenious perspective distortion to compensate for it. The coffers of the Pantheon, disposed in 5 horizontal rows of 28 coffers each, those of the first 4 rows recessed in four steps from the inner face of the dome, those of the highest row recessed in three steps, are distorted so that they refer to a vanishing point that is not located at the center of the virtual sphere, but rather at a point above the center of the pavement that coincides with the eye level of the spectator. Thus, though he is actually standing far below the spring line of the dome, it appears visually as though he is at its center (Fig. 5) [26]. The horizontal spacing of the coffers in 5 rows is interesting in itself because it models the angles of the sun at significant moments of the solar year. The angle between the center of the uppermost row of coffers and the pavement is equal to the altitude of the sun at the summer solstice; that between the center of the lowest row of coffers and the pavement, to the altitude of the sun at the equinoxes. The intermediate rows of coffers are equally spaced between the upper and lower limits.

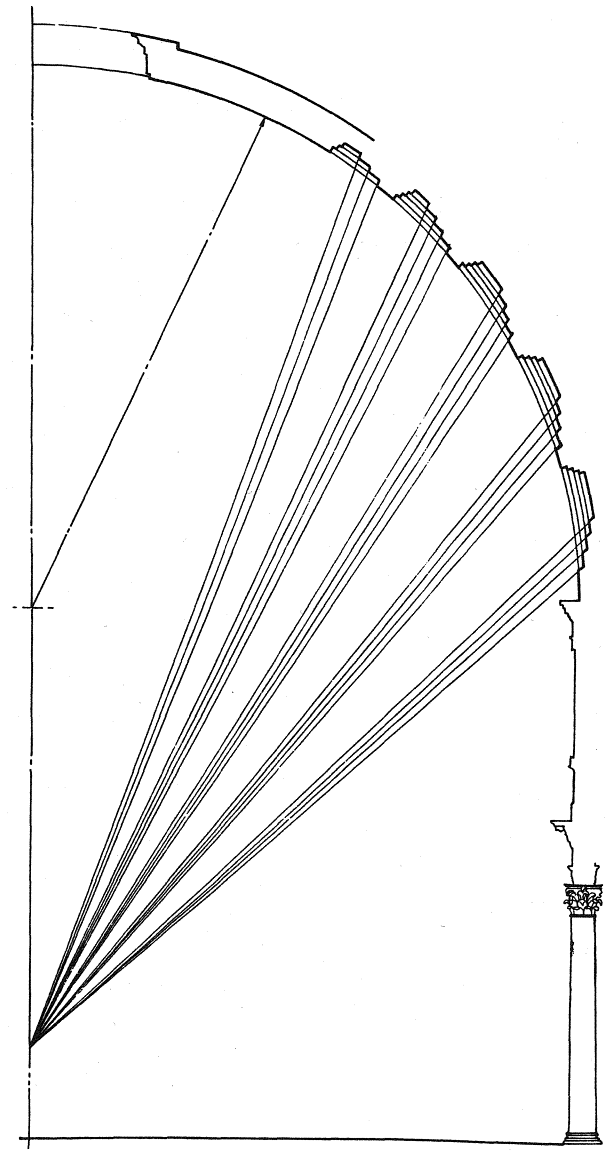

Figure 5. *The coffers of the Pantheon dome are distorted so that they appear perfectly centered from the spectator's point of view.*

Finally, the integration of diverse geometrical figures in the plan of the Pantheon brings us up to the age of Archimedes. Most Pantheon scholars agree that the basis for the architectural design was a combination of geometric figures, although there is not any general agreement as to the exact combination. By examining the elements of the Pantheon, it is possible to construct circles, square, octagons and sixteen-sided polygons, alternately inscribing and circumscribing them to create various arrangements. One such arrangement, adapted from the theory of Hermann Geertman is shown in Fig 6. This nesting of figures within and without circles may model the method of Archimedes for calculating *pi*, in which he compared the areas polygons with increasing numbers of sizes to the areas of circles which they circumscribed.

6. Conclusion

By the time of the Pantheon's construction in the second decade of the second century after Christ, the Romans not only possessed a reasonably correct picture of the cosmos, but had also mastered the engineering techniques that allowed them to construct an image of that cosmos. Alas, after the decline of the Roman Empire, both the cosmic vision and the engineering know-how was lost, only to be re-discovered a millennium later. The Pantheon had a great influence on Renaissance architects, providing not only a physical example of the suggestive power of centralized plans, but also the necessary weight of classical authority. Thus we hear from Leon Battista Alberti that roofs of temples should be vaulted and decorated with coffers, like the most beautiful of the ancient temples, the Rotunda in Rome [27]. In the sixteenth century, Renaissance architects were experimenting with the possibilities of centralized plan for sacred buildings, and the Church was engaged in a fierce battle to maintain her authority, which was threatened by Copernicus' heliocentric model of the cosmos. In the seventeenth century, the circle had been "eclipsed" by the ellipse in architecture. Is there a connection between the application of the ellipse to sacred architecture and the publication of Kepler's first law of planetary motion in 1632, which said that the orbits of the planets were elliptical? That will be the subject of my future research.

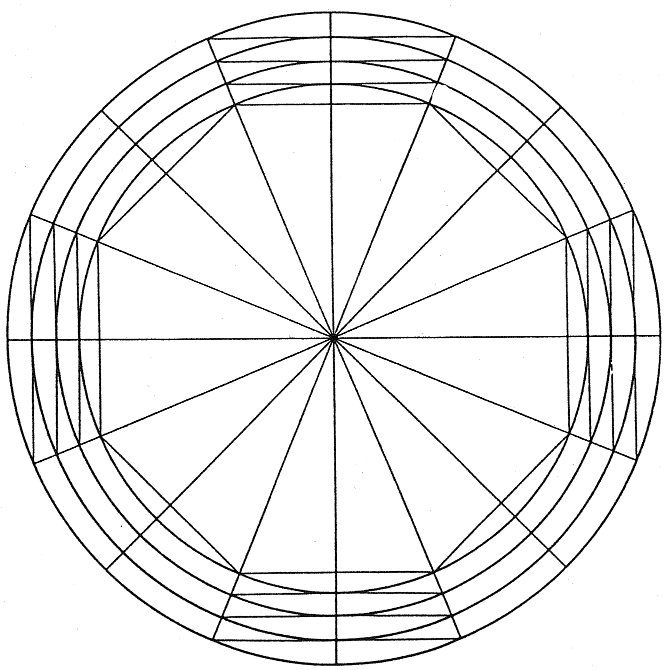

Figure 6. *One geometrical analysis of the Pantheon showing polygons nested in circles, after Hermann Geertman.*

Notes

[1] Quoted in Boyer, *A History of Mathematics,* 23.

[2] For a study of how Stonehenge functions as a predictor of celestial events, cf. Gerald S. Hawkins, *Stonehenge Decoded* (New York: Doubleday, 1965).

[3] It is interesting to note that though the Babylonians possessed what might have constituted a theoretical beginning of geometry (which will be discussed in the next section), they were not actually interested in geometry other than as a way of deriving numerical approximations. Cf. Boyer, *A History of Mathematics,* p. 38.

[4] This general statement was true up to our own century, anyway. A stunning departure from this rule can be seen in the architecture of Coop Himmelb (l) au, which relies on sophisticated computer techniques to translate directly from a sketch to working drawings, without necessitating a geometricized middle ground. See in particular the Dresden cinema complex, about which Bruno Zevi has written "...there are neither horizontals nor

verticals, and not even parallel lines (or) right angles". Cf. Bruno Zevi, "Oltre la materia", *L'Espresso,* 20 August 1998, p. 151.

[5] Cf. Robin Evans, *The Perspective Cast: Architecture and its Three Geometries* (Cambridge, MA: MIT Press, 1995) p.18.

[6] For the universality of the circle, cf. Reza Sarhangi and Bruce D. Martin, "The Circle: A Paradigm for Paradox", *Bridges: Mathematical Connections in Art, Music and Science. Conference Proceedings, 1998.* Reza Sarhangi, ed. (Winfield, Kansas: Southwestern College, 1998), 93-111.

[7] The planets of the earlier Greeks were Earth, Moon, Sun, Venus, Mars, Saturn, Neptune, and Mercury. Venus was thought of as two planets, the morning star, Phosphoros and the evening star Hesperos. Cf. Arthur Koestler, *The Sleepwalkers* (London: Penguin-Arkana, 1989), p. 49.

[8] "First, then, then, the gods, imitating the spherical shape of the universe..." Plato, "Timaeus" in *The Collected Dialogues,* Edith Hamilton and Huntington Cairns, eds., Bollingen Series LXXI (Princeton: Princeton University Press, 1989) section 44d, p. 1173.

[9] Ibid.

[10] Cf. Boyers, p. 169. A more formal statement of Plato's question is, "What are the uniform and ordered movements by the assumption of which the apparent movements of the planets can be accounted for?" Cf. *Dictionary of Scientific Biography,* Charles Gillespie, ed., New York: Charles Schribner's's sons, 1981), vol. 2, p. 28.

[11] Budoxus' model is described in Aristotle's *Metaphysics,* Book 12, section 8.

[12] "when it comes to the proportions of....the other starts to....one another, do you not suppose that he (the astronomer) will regard as a very strange fellow the man who believes that these things go on forever without change or the least deviation...? Cf. Plato, "The Republic", *op .cit.,* VII:530b, p. 762.

[13] "...our Helenic world, as I may fairly say, habitually charges high gods, sun and moon, falsely...we say that they, and certain heavenly bodies associated with them, never keep to the same path, which is why we call them planets...The fact is, friends, that the belief that the sun, moon and other heavely bodies are "wandering stars" of any sort is not true. The very reverse is the truth – each of these bodies always revolves in the same orbit and in one orbit, not many, for all that it looks to be moving in several..." Cf. Plato, "Laws", *op. cit.,* VII:821.b-822a, pg. 1391.

[14] Cf. Carl Boyer, *A History of Mathematics,* p. 11.

[15] Those who are familiar with the Pythagorean musical scale will recognize the ratio of 8/9 as the ratio of the whole tone.

[16] R. Furneaux Jordan has argued that the Parthenon of the Athenian Acropolis is a "deceptively rectangular building" and is actually comprised of circle, ellipse and parabola, but this indicates to me a knowledgeable use of geometry in order to achieve a desired visual result rather than a use of circular forms in their own right. Cf. Jordan, *A Concise History of Western Architecture,* (New York: Harcourt, Brace and World, 1971), p. 40.

[17] Cf. Mario Salvadori, *The Art of Construction,* (Chicago: Chicago Review Press, 1990) p. 111.

[18] Cf. Ibid., p. 89-90.

[19] Cf. Jordan, *op. cit.,* P. 50; Rudolph Arnheim, *The Dynamics of Architectural Form* (Berkeley: University of California Press, 1977), p. 242. Note that Jordan attributes the saying to the Arabs, Arnheim to the Hindus.

[20] Alberti makes this comparison, "But if a great number of equal arches meet at the top exactly in the center, they constitute a vault like the sky, which therefore we call the hemispherical or compleat cupola." Cf. Leon Battista Alberti, *The Ten Books of Architecture,* 1755 (rpt. New York: Dover, 1986), Book III, chap. xiv, 58.

[21] Cf. Bannister Fletcher, *A History of Architecture on the Comparative Method*, 12th ed., (New York: Charles Scribner's Sons, 1945), p. 157.

[22] The danger of constructing scientific and philosophic ideas as a historical narrative in the interpretation of a later architectural monument is that of erroneously assuming that the later period had knowledge of the earlier period. Fortunately, we have the *Ten Books on Architecture* written by Marcus Pollio Vitruvius in the first century BC, the only treatise on architecture that has survived form antiquity to the present day. He describes all aspects of the practice of architecture, including the knowledge of mathematics and astronomy that I have outlined. His writings show that he is familiar with the philosophy of the Pythagoreans and Plato; with the astronomy of Philolaus, Eudoxus and Apollonius; with the estimates of the circumference of the earth of Aristarchus and Erastothenes, therefore, with circular and spherical areas. In discussing the technique of leveling with water, he acknowledges the sphericity of the earth. He acknowledges the importance of interdisciplinary studies, and aspects of mathematics and astronomy are recorded where he feels them indispensable to the architect; the most important application of this knowledge applies to the making of sundials, which he places in the architects' repertoire. Thus he describes a geocentric universe and the circular movement of the seven planets about the earth. He specifically prescribes rules for the construction of circular temples and vaulting, an indication that by the first century this kind of building was mature.

[23] Marco Pelletti, "Note al Rilievo del Pantheon", Ouaderni d'Istituto di Storia dell'Architettura, vol. 13, 1989, pp. 10-18. Pelletti.

[24] Cf. R. Furneaux Jordan, *op. cit.,* p. 56.

[25] Gert Sperling, *Das Pantheon in Rom: Abbild un Mass des Kosmos* (Munich: Ars Una Verlag, 1999). In print.

[26] Cf. Robin Evans, *op. cit.,* p. 12.

[27] Cf. Leon Battista Alberti, *op. cit.,* Book VII, chap. 11, p. 151.

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