BRIDGES
Mathematical Connections
in Art, Music, and Science

Math and Metaphor

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Permit me to begin with the outrageous claim staked out in the title of this paper. That there is a relationship between the structure of mathematics and the structure of metaphor. By beginning here we will start very near the end of my more than decade long quest to understand the nature of language. For a student of literature, and narrative in particular, this is tantamount to ignoring the first four acts of a classical five-act drama. An additional assertion, however, is that it is ultimately nature's complex unraveling which has brought us to this point. I hope to suggest some of the analytical unraveling *(denouement)* required to understand this relationship. While I will do no more than suggest that particle physics is deeply implicated in this work, before the work is completed, metaphor, complex number, and quark will be inevitably involved.

This relatively simple conclusion has been purchased through the initial pursuit of what is called the sign by language theorists. The assumption undergirding sign theoretical work during this century is that there must be some fundamental building block of language. *Semiotics* and *deconstruction* are only the latest fields to join the hunt. Journals throughout the century are filled with efforts to determine and to model this elusive structure. In some measure my presence here as well as yours has been determined by this quest. Once led into mathematics through the study of particle physics, Nat Friedman's conferences in New York became one of several I attended. In the company of a former child prodigy in mathematics, we decided that we could do no less than host a similar conference in the Midwest. With the discovery of Reza Sarhangi, we at last had the energy and vision to undertake the conference. Without Reza, who brings not only an interest in drama, graphic arts experience, and a love for Persian poetry, we would not be gathered together. It is such a tiny gesture to thank him publicly for his passion and his wisdom, his energy and his vision of what this conference could be. And to say, as do all good prefaces, he is not responsible for any mathematical foolishness I have committed in this article.

Before I offer a few ideas to support this most absurd of conclusions, permit me to suggest a few effects which such a relationship among math, metaphor, and matter might provide:

1.) As we have partially done at Southwestern College, we can begin to move mathematics from the periphery of the university to its vital center. Or end the notion that mathematics should remain a wholly owned subsidiary of the natural sciences.

2.) We can begin to remodel curricula from kindergarten to graduate schools with a reality that once understood will provide the foundation for mathematics, for language, and for material existence.

3.) We can begin the daunting task of providing both mathematics and language a reference in the material world. Whether this will solve the problems which currently vex the study, albeit not the production, of mathematics or of language I don't know. But for those of you, who do not know there is a crisis confronting each field, you have not been paying attention.

4.) We can deepen and enrich the environmental efforts which too often have turned to mysticism or Eastern philosophy for the aid which Western rationalism has not provided.

The problems which assail such an effort with regard to the foundation of mathematics issues are, at first glance, overwhelming. The question in many forms is still best stated by Herman Weyl in 1946 [1]:

> The question of the ultimate foundations and the ultimate meaning of mathematics remains open; we do not know in what direction it will find its final solution or even whether a final objective answer can be expected at all. "Mathematizing" may well be a creative activity of man, like language or music, of primary originality whose historical decisions defy complete objective rationalization.

While I appreciate any set composed of mathematics, language, and music, the familiar belief that a foundation does not exist or cannot be discovered seems here to be based in a presupposition that finds creativity a synonym for anarchy or at best causeless existence. The very fact that music, language, literature, and mathematics are rule-governed activities should at least cause us to continue our search for the fountainhead of those rules. The relationship of mathematics and metaphor will bring us closer to a foundation than has either logic or formalism in the study of math.

W.V.O. Quine, a logician, raises the question: "What if any thing is mathematics about?" Quine goes on to wonder, "How do we ever know anything? We seem to bump into things in the dark." At a symposium on complexity, entropy and information held by the Santa Fe Institute, we have the problem framed from the physical point of view. P.C. Davies writes: "It has to be concluded that, at this time, the answer to the question 'Why is the universe knowable?' is unknown. The dazzling power of mathematics to describe the world at a basic level continues to baffle us. The ability of a subset of the universe (the brain) to construct an internal representation of the whole, including an understanding of that basic level, remains an enigma." [2]

Another approach to the size of the problem was once undertaken by Stanislaw Ulam. He refers to it in his autobiography, *Adventures of a Mathematician*.

> At a talk which I gave at a celebration of the twenty-fifth anniversary of the construction of von Neumann's computer in Princeton a few years ago. I suddenly started estimating silently in my mind how many theorems are published yearly in mathematical journals. I made a quick mental calculation and came to a number like one hundred thousand theorems a year. I mentioned this and my audience gasped. The next day two of the younger mathematicians in the audience came to tell me that, impressed by this enormous figure, they undertook a more systematic and detailed search in the Institute library. By multiplying the number of journals by the number of yearly issues, by the numbers of papers per issue and the average number of theorems per paper, their estimate came to nearly two hundred thousand theorems a year.

Ulam's conclusion to this overwhelming production was that:

> The judgment of value in mathematical research is becoming more and more difficult, and most of us are becoming mainly technicians. The variety of [mathematical] objects worked on by young scientists is growing exponentially. Perhaps one should not call it a pollution of thought; it is possibly a mirror of the prodigality of nature which produces a million species of different insects [3].

John von Neumann observed in the 1940's that a skilled mathematician might know ten percent of available mathematics. Perhaps true then, if only for someone of von Neumann's skill and phenomenal memory. Today few if any would attempt to quantify knowledge within mathematics. Gone too is the belief that applied mathematics provides the true explanation for a physical event. Thus theorems and proofs are accepted or rejected on the basis of qualities such as simplicity (law of least action) or the practical judgment of fruitfulness. Philip Davis and Reuben Hersh note and lament the loss of certainty in mathematics study [4]:

> Courses in increasing number are being taught under the name "Mathematical Modeling." What would have been taught as "the theory of such and such", now is known merely as "model for such and such." Truth has abdicated and expediency reigns.

Apparently a small but growing number of mathematicians, who have shown an interest in - what we often viewed suspiciously as philosophical issues - suspect that no definitive answer will be found to the foundation of math dilemma. They have made the decision to exit the foundation search for more pragmatic investigations. What constitutes progress in mathematics? Why and how does mathematics grow? Is mathematics evolutionary or revolutionary? Why do certain fields prosper and others disappear or become exhausted? From our vantage point we can easily understand the desire to escape the vicious or the hermeneutic circle for something closer to a sociology of math, i.e. mathematics' role in a human community. It is just as obvious, however, that each question followed far enough will inevitably return us to the foundation problem. There we will be forced to discover reference in the material world or to turn inside mathematics or the mathematics sign system to create one more "bootstrap" theory of mathematical truth.

In making the case for investigation outside the vexing foundational issue, mathematical historians Aspray and Kitcher write [5]:

> Philosophy of mathematics appear to become a microcosm for the most general and central issues in philosophy - issues in epistemology, metaphysics, and philosophy of language - and the study of these parts of mathematics to which philosophers most often attend (logic, set theory, arithmetic) seems designed to test the merits of large philosophical views about the existence of abstract entities or the tenability of a certain picture of human knowledge.

Philip Kitcher [6] extends this analysis with a distinction perhaps more familiar to historiographers than to mathematicians:

> Epistemic justification of a body of mathematics must show that the corpus we have obtained contributes either to the aims of science or to our practical goals. If parts can be excised without loss of understanding or of fruitfulness, then we have no *epistemic* warrant for retaining them. If there is a distinction between mathematics as art and mathematics as cognitive endeavor, it is here that it must be drawn.

The hoped for distinction, however, is at best elusive and, at worst, non-existent.

Had we not seen it all before, the effort to separate art and "cognitive" endeavor would be amusing. Imagine the author's surprise to learn that the issues he raises about inquiry into mathematics are also issues relevant to art, music, and literary analysis.

The major distinction is between those outside the analysis of art and those inside. The former knows instinctively what art is. Those who work within analysis of literature, art, or music don't have this luxury. Unfortunately, few of these raise questions about the nature and use of mathematics, for both obvious and little understood reasons. The decision to set art aside in the study of mathematics and language was made by C.S. Peirce and Ludwig Wittgenstein. Some day we will understand the tragic dimensions of that decision.

Thus, trapped between those who would find meaning in pragmatic versions of language games or conventions and those who would seek mathematics' role as a field subservient to science (or worse, to engineering), many mathematicians retreat to the safety of isolated purity. Here math is about nothing other than itself. Few would "demean" the field by calling it art. Others have explained the phenomenon by suggesting that mathematicians who plead purity are philosophically unaware or disinterested. Some continue to hold the thoroughly discredited idea that all of mathematics is a version of formal logic and therefore teaches students how to think clearly.

While the belief that mathematics is about itself may provide some protection from the philosophical and scientific despoilers, it nonetheless illustrates the vulnerability of a field without an identity in reference. We may conclude that this is either similar to or identical with the problem facing language without eliminating the problem itself.

As for the valuable issue of whether mathematics is invention or discovery, most working mathematicians simply turn away. One response from a one time member of the Board of Governors of the Mathematical Association of America, Jerry King, is that "On Tuesdays and Thursdays my colleagues and I think it is invention; on Mondays and Wednesdays - discovery. But usually we don't think about the problem at all." And thus we have another echo of Quine's rhetorical question, "What if any thing is mathematics about?" And of the significance of that question to working mathematicians.

To those who continue to work on the problem, many are too ready to concede that mathematics is simply one more art form. This concession before we understand the nature of art or of creativity, comes far too quickly.

One illustration of this concession comes from mathematicians Philip Davis and Reuben Hersh. Reviewing the various approaches to the foundation quest, they too are ready to give up the search. With efforts to ground mathematics in logic at a virtual end, Gödel's Incompleteness Theorem attracting more attention from surprised computer scientists than from philosophers, and the displacement of truth by the concept of modeling, the quest for a foundation seems to have been conceded to psychobiology. This is not a surprising retreat to those who have followed linguistics' retreat to psychobiology for those elusive deep structures once promised us by Noam Chomsky. For Davis and Hersh, psychobiology comes in the form of intuition. The difficulty with intuitionism in mathematics is in the ambiguity produced precisely because no one knows what intuition is. Davis and Hersch offer six possible meanings which range from "the opposite of rigorous" to "holistic or integrative." More important is their conclusion regarding intuition in mathematical reasoning.

> We maintain that
>
> (1) All the standard and philosophical viewpoints rely in an essential way on some notion of intuition.
>
> (2) None of these even attempt to explain the nature and meaning of the intuition which they postulate.
>
> (3) A consideration of intuition as it is actually experienced leads to a notion which is difficult and complex, but it is not inexplicable or unanalyzable. A realistic analysis of mathematical intuition is a reasonable goal, and should become one of the central features of an adequate philosophy of mathematics. [7]

Although I agree with each of these three points, Davis and Hersh are forced to concessions, which are unacceptable. Their effort to demonstrate that intuition is "not inexplicable or unanalyzable" leads to one more form of pragmatism or cultural language game. This is territory already explored by Wittgenstein, Peirce, and many others. It remains a bold concession on the part of practicing mathematicians, but does not advance the search for a foundation.

While Davis and Hersh had already noted the retreat from certainty in the shift from "theory of" to "model of" in course descriptions and elsewhere, they are powerless to reverse this change. They write [8]:

> Mathematics does have a subject matter, and its statements are meaningful. The meaning, however, is to be found in the shared external nonhuman reality. In this respect, mathematics, is similar to an ideology, a religion, or an art form; it deals with human meanings, and is intelligible only within the context of culture. In other words, mathematics is a humanistic study. It is one of the humanities.

By this point, the delivery of mathematics into the camp of the humanities is understandable. The assumptions about ideology, religion, and art are all too familiar. We will pause only to wonder what exists outside the context of culture? Of course the authors intend, as did Wittgenstein, to leave scientific propositions as examples of "an external nonhuman reality." Meanwhile the goal of understanding intuition remains. The question of what math is about, if any thing, cannot be fully pursued without addressing the issue of intuition. While such a survey is not possible here, I will suggest the truth is more complex than many of those exploring intuitionism in mathematics even suspected. It should be obvious to anyone who chooses invention over discovery that in the application of complex numbers or in the discoveries of fractal geometers that we have irresistible proof of Plato's, Galileo's, and Poincare's sense of mathematics. Namely that some heretofore unknown reality is being discovered in some of the mathematical modeling done. In saying this, I am well aware of the many challenges to quantum reality and also of Stephen Smale's challenge to Benoit Mandelbrot over the issue of "robustness". That not even fractals are sufficiently strong to reflect the material world. I would also grant E.T. Bell, Davis, Hersh, and others that "invention" applies more often to mathematical labors than does "discovery."

The Mysterious Complex Number

British mathematician Roger Penrose was among the earliest to recognize a major implication of Benoit Mandelbrot's work. He says, "It would seem that this structure [the Mandelbrot set] is not just part of our minds, but it has a reality of its own." Later he adds force to the observation: "The Mandelbrot set is not an invention of the human mind: it was a discovery. Like Mount Everest, the Mandelbrot set is just there!" On the issue of discovery versus invention, Penrose had some time ago declared himself on the side of the existence of mathematical structures whose reality is not dependent on culture or creativity. His work with physicist Stephen Hawking and with his students on four-dimensional objects in the realm of complex analysis had prepared him to declare himself now as an advocate of discovery [9]:

> When mathematicians come upon their results are they just producing elaborate mental constructions which have no actual reality, but whose power and elegance is sufficient simply to fool even their inventors into believing that these mere mental constructions are "real"? Or are mathematicians really uncovering truths which are, in fact, already "there" - truths whose existence is quite independent of the mathematicians' activities? I think that, by now, it must be quite clear to the reader that I am an adherent of the second, rather than the first, view, at least with regard to such structures as complex numbers and the Mandelbrot set.

David Peat has offered one of the more concise, albeit fanciful, explanations of this work in the complex plane [9]:

> The propulsion system that is jetting the computer into the Mandelbrot set is the equation $z + c$. z is a complex number allowed to vary and c is a fixed complex number. The adventurer sets his or her two complex numbers into the equation and tells the computer to take the result of the addition $z + c$ and substitute it the next time around (and the next time around after that (. . .) for z). [10]

Here, the three dots within parens is not a notation for an ellipsis, but the mathematical notation for iteration, i.e. the repetition of an operation.

Complex numbers have historically been referred to as constructs dependent on "impossibles" or "imaginaries." Long before complex analysis arrived, however, math had struggled with other forms which challenged a sense of concrete reality. The history of the zero is one such example and contributes to some of the semantic problems which have challenged philosophers and historians. For us to deny the use of the zero would be foolish. But the greatest mathematicians of the classical world did just that. Without the zero we know now we would not have had algebra or most forms of mathematics which have contributed to our intellectual and technological progress. Many credit the discovery (invention) of the zero with being among the greatest single achievements of humankind. Without the principle of position, achieved by the zero, or the ability to generalize the number concept, the development of science and industry would be hard to imagine.

But to say this is certainly not to say that our achievements stand on an abyss, or quick sand, or nothing. Tobias Dantzig, math historian, frames the issue by wondering why the Greeks, who did such extraordinary work with geometry, did not develop a simple algebra. "Is it not equally strange," he asks, "that algebra, that cornerstone of modern mathematics, also originated in India and at about the same time when positional numeration did?"

Dantzig answers his question with a brief discussion of the counting board. Here he finds that the counting board would be hopelessly ambiguous if there were no way to symbolize the gaps or empty columns. If we represent 32, for example, with three marks followed by two marks (≡ =), many numbers could be inferred. The only way to stabilize position is to create null columns. Then 32 becomes

			≡	=

and 3020 is

	≡		=	

We see therefore that no progress was possible until a symbol was invented for an empty class, a symbol invented for nothing, our modern zero. The concrete mind of the ancient Greeks could not conceive the void as a number, let alone endow the void with a symbol. And neither did the unknown Hindu see in zero the symbol of nothing. The Indian term for zero was *sunya*, which meant empty or blank, but had no connotation of "void" or "nothing." [11].

The conflicting attempts to fix the nature or the reference of the zero is repeated in the mathematical history of the West. We first see the problems caused by irrational numbers captured in all the retelling of the Pythagoreans' efforts to protect the whole number as the basis of all reality. Then the parallel struggle to protect mathematics from the void or the zero. Next comes the perplexing problems raised by negative numbers. And finally the baffling difficulties created by complex numbers. Kline acknowledges that by the sixteenth century, the zero had at last been accepted as a number and irrational numbers were in use. He quotes Michael Stifel who, in *Arithmetica Integra* (1544), admits to the practical aid irrational numbers have provided. On that basis Stifel considers accepting them (the square root of three, for example) as true numbers. But he remains troubled [12]:

> On the other hand, other considerations compel us to deny that irrational numbers are numbers at all. To wit, when we seek to subject them to numeration [decimal representation]. . .we find that they flee away perpetually, so that not one of them can be apprehended precisely in itself. . . . Therefore, just as an infinite number is not a number, so an irrational number is not a true number, but lies in a kind of cloud of infinity.

Negative numbers did not fare any better. Sixteenth and seventeenth century mathematicians generally rejected them, either as numbers or as roots of equations. Some spoke of them as absurd, with both the technical and lexical meaning implied. Others called them impossible solutions, symbols, or fictions. Because negative roots could be transposed into positive roots, Descartes accepted the negatives as numbers. Blaise Pascal, however, found the subtraction of a whole number from zero a nonsensical operation. And Kline notes, "Without having fully overcome their difficulties with irrational and negative numbers the Europeans added to their problems by blundering into what we now call complex numbers." [13].

The history of complex numbers is filled with low and high comedy. In the face of the ridicule and the rejection of the complex numbers, Albert Girard wrote in 1629, "One could say: Of what use are these impossible solutions [complex roots]? I answer: For three things - for the certitude of the general, for their utility, and because there are no other solutions." Descartes accepted negative roots but rejected complex roots because transformation was not possible. He said, "Neither the true nor the false [negative] roots are always real; sometimes they are imaginary." And "imaginary" numbers they have remained. Newton showed little interest in them other than as indicators of non-physical or non-geometrical solutions. Leibniz characterized them as "that amphibian between being and not-being, which we call the imaginary root of negative unity." [14].

An effort was made to remove the stigma of the word "imaginary" when they were plotted on the y-axis or the vertical axis and the real numbers corresponded to points on the x-axis or the horizontal axis. The vertical axis was called the normal axis (since "normal" denotes a line that is perpendicular to another line) and hence the suggestion that these numbers be called normal numbers - an altogether confusing idea. And so the notation *i* appeared, to indicate imaginary and to supplant ($\sqrt{-1}$. Once paired with real numbers, imaginaries became complex numbers.

This is an obviously superficial summary of a magnificent historical development which begins in India, crosses through the Arab world into Europe, and, if pursued, a development which leads through abstract algebra, non-Euclidean geometry, number theory, economics, work in high energy physics, laser technologies, and fractal geometry.

In order to illustrate the shift in our understanding of imaginary numbers in just a few decades, here is first, one side of a dialogue between two physicists who made important contributions to atomic theory in the 1930s and beyond, Heisenberg and Bohr. The subject is the choice between religious and mathematical knowledge. Heisenberg says [15],

> Mathematicians, as everyone knows, work with an imaginary unit, the square root of -1, called *i*. We know that *i* does not figure among the natural numbers. Nevertheless, important branches of mathematics, for instance the theory of analytical functions, are based on this imaginary unit, that is, on the fact that ($\sqrt{-1}$ exists after all. Would you agree that the statement "There is a ($\sqrt{-1}$" means nothing else than "There are important mathematical relations that are most simply represented by the introduction of the ($\sqrt{-1}$ concept?" And yet these relations would exist even without it. That is precisely why this type of mathematics is so useful even in science and technology.

Heisenberg then turns to mathematical infinity and suggests that religion is also a parallel effort to capture in abstraction those connections which exist regardless of the "reality" of the language used. In a familiar gesture, Bohr dismisses math as "a mental game that we can play or not play as we choose." Religion is, for Bohr, something personal and abiding.

We should certainly qualify if not reject Bohr's analogy of math as mental game. In addition we might reverse Heisenberg's observation that "these relations would exist without [the concept of the square root of negative one]" to read: the square root of negative one would not exist without these relations, i.e. relationships in nature. It is only possible to arrive at either conclusion with any certainty at the close of this century.

The distance between Heisenberg and Roger Penrose should signal the epistemological ground we have covered as well as our return to some form of Platonism. Penrose says [16],

> While at first it may seem that the introduction of such square roots of negative numbers was just a device - a mathematical invention designed to achieve a specific purpose - it later became clear that these objects are achieving far more than that for which they were originally designed. . . . Although the original purpose of introducing complex numbers was to enable square roots to be taken with impunity, by introducing such numbers we find that we get, as a bonus, the potentiality for taking any other kind of root or for solving any algebraic equation whatever. Later we find many other magical properties that these complex numbers possess, properties that we had no inkling about at first. These properties are just there.

Only with an effort to ground the complex number in material existence will anyone be able to demonstrate that Penrose's image of "magical properties" is actually a natural and a rational property of complex numbers. Or, at least, that nature is the magician and we are assistants and audience for what is, after all, the oldest of magic acts.

The Magical Metaphor

In our efforts to discover bridges or identities among sign (metaphor), complex number, and quark we might additionally provide a service to natural scientists whose understanding of language usually ignores this century's achievements in language analysis. From Niels Bohr to twice honored Nobel laureate Sheldon Glashow, physicists write about language as though unaware that some of our finest minds have dedicated their professional lives to the examination of natural language. For example, physicist David Bohm's proposed concept for language, which he calls the "rheomode," suggests that we need to shift attention to a world in motion by emphasizing the verb-like quality of life. New labels for language are interesting, but are hardly the solution to our dilemma. The solution is in language, not in the reconfiguration of language to fit a new linguistic context. Language is arguably wiser than we are. Its evolution is rule-governed and purposeful, not perverse.

In addition, a metaphor-complex number-quark model holds the possibility of contributing to the mind-brain dialogue and to efforts to revive the quest for hard artificial intelligence in this century. Whatever the mind is determined to be it is obvious that it is ineluctably, in part or whole, language. The brothers and number theorists introduced in a March 1998 *Chronicle of Higher Education*, echo the ancient Pythogorean belief that "all is number." David Chudnovsky says, "In our opinion everything is mathematics." And Gregory Chudnovsky replies, "Or boils down to mathematics." [17]. I would agree and also repeat Gore Vidal and others who have asserted that "all is metaphor." Only at the end of this century do we have the possibility that these are not contradictory assertions.

Umberto Eco reviews the complicated nature of metaphor by drawing together the etymological history with current linguistic and computer program work. His conclusion opens again the possibility that continental language philosophy and semiotic analysis will some day be joined:

> At any rate, for too long it has been thought that in order to understand metaphors it is necessary to know the code (or the encyclopedia): the truth is that the metaphor is the tool that permits us to understand the encyclopedia better. This is the type of knowledge that the metaphor stakes out for us. [18].

In the familiar reductionism of our age, metaphor is often presented by high school texts and literary dictionaries as "a comparison between two unlike things without the use of like or as". This paradoxical definition alone contains much of the mystery and productivity of our species. Vico called metaphor "the most luminous and therefore the most necessary and frequent" of all figures of speech. The Venerable Bede said that metaphor is "a genus of which all the other tropes are species." In our time, Kenneth Burke suggests that metaphor offers a "perspective": "Metaphor is a device for seeing something in terms of something else. . . . A metaphor tells us something about one character considered from the point of view of another character. And to consider *A* from the point of view of *B* is, of course, to use *B* as a perspective upon *A*." Bedell Stanford describes metaphor as "the stereoscope of ideas" which provides an "integration of diversities."

A 1971 bibliography of metaphor records approximately 3,000 titles by overlooking most of literary analysis which could not function without reference to metaphor as well as renewed interest in the social sciences generally and in cultural anthropology particularly. For example, the subtitle of Joseph Campbell's final work before his death was titled *Metaphor as Myth and as Religion*. Aristotle observed that the "greatest thing by far is to be a master of metaphor. It is the one thing that cannot be learned from others. It is the mark of genius." And Umberto Eco notes that it "defies every encyclopedic entry." [19].

What we have been taught to ignore is the dynamics of the metaphor. By seeing the figure as a simple two part system we have missed its power. This is of course a recapitulation of the discrete/continuous debate which continues to affect discussions of mathematics. We may choose to emphasize the continuous or the discrete, but both are ineluctable features of the metaphor. It is possible to view metaphor as a simple comparison, involving a suppressed "like" or "as" as a hidden term. "No man is [like] an island." The cloud is [like] cotton candy. Despite Greek, Roman, and medieval philosophers' best efforts to create taxonomic systems for this trope, the results are at such a high level of abstraction that they are useless or obviously reductive even within the few distinctions they make. Or they are so phenomenally complex that the very point of taxonomy is called into question. Metaphor is the equivalent of mathematics' elusive set of all sets.

When obvious metaphors appear in contemporary math or science texts, usually the simplest of comparisons is intended. This of course ignores the perspectives that Rexroth discusses or the stereoscopic effect in an "integration of diversities" which Bledsell emphasizes. If there is quantum weirdness, the popular phrase for the waves versus particles debate-called paves or wavicles in some circles, we may find it more often in the physicists' patient explanations of the stereoscope, or of the *trompe l'oeil* exercises-now a woman, now a vase-or in the fascination with contemporary artists who offer dualism as structural features of their work. Metaphor has long known what physicists have so recently discovered.

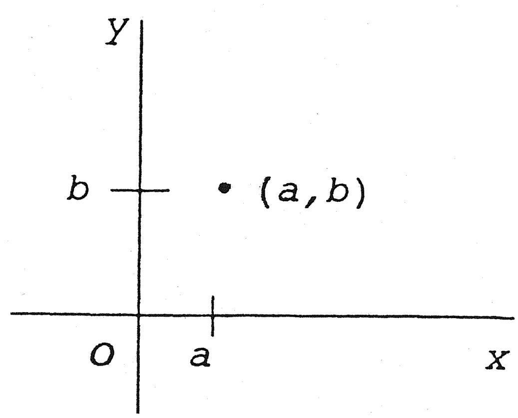
We may understand this stereoscopic vision or integration as a superimposition of one effect on another. (A parallel of the important concept of superposition in subatomic physics.) Or we may view it as an assertion of identity, which reveals a single similarity (or identical feature), a collection of similarities (or identical features), or, at its most extreme, an absolute fusion of the elements. It is only a judgment of Western rationalism which focuses on metaphors as having two parts. In order to explain the creative potential of metaphor, some speak of it as the union of two formerly discrete elements which produces a third. (We should point out that no single word is ever truly discrete. Only the loss of departments of etymology or of historical linguistics would even permit us to consider this a possibility.) But to keep us above the foam which genuine analysis of language produces, we may continue the illusion that two parts are brought together to produce a third. Few creative writers would challenge this concept of creativity. But then neither would an obstetrician or a pediatrician. Is the created third discrete or continuous?

Poet Robinson Jeffers wrote, "Violence has been the sire of all the world's values." I would insist that he didn't mean to say, "Violence has been like the sire." Had he intended the simile, he would have used it. In Native American naming and healing rituals the intention is not to suggest a fragile similarity between human and nature. As irrational as scientific analysis finds it, identity is being asserted. In other forms, it is the *tat tvam assi* (thou art that) of Hindu mysticism or "this is my body, this is my blood" of Christian communion. The great transubstantion versus consubstantiation debate within the church indicates just how problematic this paradox (contradiction from a rationalist point of view) has been.

While many have commented on the presence of contradiction in mathematical systems, Henri Poincare's observation remains timely. He says of efforts to axiomatize set theory, "We have put a fence around the herd to protect it from the wolves but we don't know whether some wolves were already inside the fence." [20] The wolves of paradox/contradiction have forever assailed mathematics. The debate can be traced from the early intuitionist attack on classical syllogisms to the paradox riddled calculus which so frustrated Alan Turing to the difficulties created by complex numbers used by high energy physicists. Only Stephen Hawking had enough curiosity to begin to suspect that these infinities physicists kept getting, and which were routinely eliminated by a questionable procedure called renormalization, were "trying to tell us something."

In spite of the most strenuous efforts to protect the herd of mathematical systems from contradiction and from metaphysics, it is obvious by now that the wolves have always been inside the fence. Gödel has demonstrated, if the Pythagoreans had not, that paradox (inconsistency, contradiction) is unavoidable. Whitehead and others have shown that deductive reasoning is inevitably metaphysical. This after induction had been shown to be barren, incapable of producing viable hypotheses. What we are left with is a series of metaphysical explanations for our ability to create mathematics and language. The terms change as we attempt to discover the source of our ability: Eternity, Platonic realm, God, the Old One, nature, the subconscious. The mechanisms also shift: revelation, induction, deduction, abduction, hypothetico-deductive reasoning (Einstein's choice), DNA, psychobiology. The mind or the brain-mind is merely the latest locus of the quest for a foundation for mathematics and language.

Metaphor in its paradoxical form is the equivalent of a complex number, $a + bi$. In its binary form, metaphor is the equivalent of a pair of real numbers (a, b). The individual numbers a and b may be viewed as points on the respective horizontal and vertical axes of a graph. Joined, they may be seen as a single point in a plane:



The point (a, b) is a graphic representation of the complex number a + bi. What mathematicians have done, unknowingly, is to model or to reproduce the structure of metaphor. In a sense, the alpha of language and of literature is also the origin of mathematics. It is not much of a leap at all to assume that the origins of mathematics are not in the "natural numbers" but at least in part in complex numbers Wittgenstein once wrote his mentor Bertrand Russell, "Identity is the very devil." I am led to conclude that there is some deep isomorphism between the forms of matter, the complex number, and metaphor. Until we have completed this work, perhaps it is safer to just say a relationship exists among the three.

Alain onnes, holder of the chair in Analysis and Geometry at the *College de France*, arrives at a similar conclusion with regard to the origin of mathematics. Arguing for what he calls "archaic mathematical reality," Connes says "...one of the main unsolved problems involves developing the right geometric concepts for understanding a space that mathematicians call the arithmetic side. The geometric understanding of this set should contain many of the regularities observed in the sequence of prime numbers. It's by no means unimaginable that the new geometry required by physicists for the understanding of quantum gravity will turn out to be the same as the one required by mathematicians for the understanding of the arithmetic site. If this turns out in fact to be the case, it would go a long ways to demonstrating how intricate the relation between physical reality and what I like to call 'archaic mathematical reality' really is." [21]

When we have achieved a unified theory in physics we will be closer to unification in mathematics and in language. I share the dream of mathematical unification with a few others who believe that should this monumental task be achieved, unification in particle physics and in language theory will be assured.

References

[1] Quoted in Morris Kline, *Mathematical Thought*, vol. 3, 1210. See also Rolin Wavre, "Is There a Crisis in Mathematics?' *American Mathematical Monthly*, vol. 41. No. 8 (October 1934), 488ff. Reproduced, in part, in Edna E. Kramer, *The Nature and Growth of Modern Mathematics* (Princeton: Princeton University Press, 1982), 692-696.

[2] Wojciech H. Zured (ed.) *Complexity, Entropy and the Physics of Information* (Redwood City, California: Addison-Wesley Publishing Company, 1990), 68.

[3] In Philip J. Davis and Reuben Hersh, *The Mathematical Experience* (Boston: Houghton Mifflin 1981), 20-21. Originally in, Stanislaw Ulam, *Adventures of a Mathematician* (New York: Scribner's 1976). Davis and Hersh refer to the annual volume of mathematics as "Ulam's dilemma." While Ulam contended that most mathematicians are forced to become "mainly Technicians," he does conclude his field commentary on an optimistic note. Hersh and Davis are considerably less optimistic: "unavoidable problems of daily mathematical practice lead to fundamental questions of epistemology and ontology, but most professionals have learned to bypass such questions as irrelevant." (23)

[4] Davis and Hersh, 70.

[5] William Aspray and Philip Kitcher, eds., *History and Philosophy of Modern Mathematics* (Minneapolis: University of Minnesota Press, 1988), 17. Davis recognizes this point of view and argues for a radical alteration in the teaching of mathematics. He proposes that mathematicians study the "semantics" rather than the "syntax" of mathematical applications: "The technical term for inquiries such as the above is 'hermeneutics.' This word is well-established in theology, and in the last generation has been commonly employed in literary criticism. It means the principles or the lines along which explanation and interpretation are carried out. It is time that the word is given a mathematical context. Instruction in mathematics must enter a hermeneutic phase. This is the price that must be paid for the sudden, massive and revolutionary intrusion of mathematizations-computerizations into our daily lives." From, "Applied Mathematics as Social Contract," *Mathematics Magazine*, vol. 61, no. 3 (June 1988), 146.

[6] Kitcher, "Mathematical Naturalism," *History and Philosophy of Mathematics*," 315.

[7] Davis and Hersh, *The Mathematical Experience*, 392.

[8] Davis and Hersh, 410.

[9] Roger Penrose, *The Emperor's New Mind: Concerning Computers, Minds and the Laws of Physics* (New York: Oxford University Press, 1989), 95.

[10] John Briggs and F. David Peat, *Turbulent Mirror: An Illustrated Guide to Chaos Theory and the Science of Wholeness* (New York: Harper and Row, 1989), 96.

[11] Dantzig, 30-31.

[12] Kline, *Mathematical Thought*, vol. 1, 257.

[13] Kline, 254

[14] Kline, 254

[15] Werner Heisenberg, *Physics and Beyond: Encounters and Conversations*, trans. Arnold J.Pomerans (New York: Harper and Row, 1971), 89. Heisenberg's understanding of religion as an earlier expression of "scientific thought" seems all but assured. Mathew Fox is one of those attempting the reunion of cosmology and religion. See, for example, *The Coming of the Cosmic Christ* (New York: Harper and Row, 1983). Someday we night understand that the scholastic theologians' question about the number of angels able to dance on the head of a pin, which we have been taught to dismiss as nonsense, was merely an early investigation of infinity. Greg Cantor read theology in order to develop his own work with set theory and infinity. Connections between Einstein's relativity theory and the Torah, quantum reality (and deconstruction) and cabalistic philosophy, and Mikhail Bakhtin's thought have been made. As Kenneth Burke said, "What we say about words in the empirical realm will bear a notable likeness to what is said about God in theology." Clark and Holquist add: "This is so ecause the inescapable dualities of theology (man/God, spirit/matter) are at the heart of language in the duality of sign/signified" (*Mikhail Bakhtin*, 83) This duality is at the heart of matter in the wave/particle or field/particle paradox. We live in one world with one set of laws which governs. The university offers a many worlds image of this reality.

[16] Penrose, 96.

[17] Vincent Kiernan, "With Abstruse Mathematics as a Tool, 2 Bothers Tackle Real-World Problems," *The Chronicle of Higher Education*, vol. 46, no.28 (March 20, 1998), A20.

[18] Umberto Eco, *Semiotics and the Philosophy of Language* (Bloomington: Indian University Press, (1984), 87. Other definitions of metaphor have been gathered from various and occasional commentaries on this trope of all tropes. It is among the most studied and least understood of literary figures. Robert Frost once wrote: "I have wanted in later years to go further and further in making metaphor the whole of thinking. I find someone now and then to agree with me, that all thinking is metaphorical; or all thinking except scientific thinking." (In "Education by Poetry," *The Dolphin Reader*, Hunt (ed.) 2nd ed. Boston: Houghton Mifflin, 22-23). By understanding the metaphor's relationship to mathematics and eventually to the thought and work of foundational physics, we will cease to be puzzled by those who find the world or the cosmos a great thought or a great poem. To dismiss these relationships as charming "metaphors," as metaphysics, or as mystical aberrations is to ignore the structural identity of thought, mathematics, and wave-particle interactions. Only when foundational unification is achieved will we begin to understand that the distinction between physics and metaphysics has been usefully specious nonsense.

[19] Eco, 129.

[20] Kline, vol.3, 1186.

[21] Jean-Pierre Changeu and Alain Connes *Conversations on Mind, Matter, and Mathematics* (Princeton, New Jersey: Princeton University Press, 1995), 208-209,