BRIDGES
Mathematical Connections
in Art, Music, and Science

Mathematics in Three Dimensional Design
The Integration of Mathematical Thinking into the Design Core

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Introduction and Background

Five years ago I became Coordinator of our School of Visual Arts freshmen level Core Design Program. Until that time, I had been teaching senior undergraduate and graduate level students in our sculpture program. Suddenly, I was surrounded by freshmen and I experienced an awakening to a new generation of "screenagers"[1]. Coincidental to my assuming responsibility for the design program, my colleagues and I came to a consensus about instituting a new instructional format to ensure greater continuity in the presentation of the program. This format now consists of one hour per week of lecture on the elements and principles of design and four hours per week of laboratory application of those elements and principles. As program coordinator I present the weekly lectures in a large class and supervise 20 graduate teaching-assistants who teach the labs under my immediate direction. Our program is large and averages 350 to 400 design students per semester. During any given semester, approximately half of those students are involved with three-dimensional design issues.

In my mind the implementation of the new lecture/lab format called for a review of the entire design curriculum. I wanted to examine the nature of student questions about the validity of continuing to teach the program according to the old teaching model, which promoted a high degree of diversity from one lab section to another. Times have changed and so have methodologies in education.

Throughout the history of my own development as an artist, I have maintained a close link with mathematics and the physical sciences. Had I not taken the path of art I would have pursued a career in the sciences. Consequently, I have always looked to mathematics and the sciences for inspiration to do my own work. The same semester I took over the core design program I implemented a new graduate level experimental course in Art and Mathematics. I collaborated with one of my colleagues in the mathematics department, a fractal geometer. I invited specific students, whom I perceived to share my interest in mathematics and the sciences, to participate in this first course offering. The enthusiasm generated by this course was overwhelming. Word of it filtered down into the undergraduate program even to the freshmen level design students. To my amazement I was approached by students of all levels and asked when I would offer such a course again. Prior to that, my experience of trying to mix art students with mathematics was like trying to mix oil with water. What does mathematics have to do with art? The fact of the matter is that mathematics has a lot to do with art, more than traditional art students have ever imagined. Artists unconsciously utilize mathematical concepts and apply practical mathematical algorithms on a regular basis in the process of their art making. To my knowledge traditional pedagogy in the arts has never emphasized the role of mathematical thinking in art making. For my students and me the time had arrived.

Mission and Methodology

In my opinion one of the primary objectives of a good foundations of art program should be the meaningful integration of mathematical thinking into the curriculum. In an age of computer technology such integration is essential to the education of a well-rounded art major. Many of our art programs have been taken over by the introduction of technology. Almost anyone can learn to operate a graphics software program without benefit of knowing the underlying mathematical concepts or mathematically related operations of the computer. My motivation for advocating mathematical thinking in three-dimensional design, however, is much more fundamental; it stems from my thesis that the most effective way to teach the elements and principles of design is from a historical context. By providing students with a historical narrative back drop for the introduction of certain design project activities, which seem to have no practical value, students are inherently less inclined to question the validity of such projects and are willing to participate without resistance. My methodology then is to build a sound three-dimensional course of study based on the history of mathematics and illustrate its relationship to the development of design thinking.

Due to the limits of time and space I will not be able to lead the reader through an entire semester of this methodology. I will, however, present the first few phases of this mathematical approach to design. Assuming that we can all agree that the elements of design are line, shape, space, texture, value, and color and that the principles of design are harmony, variety, balance, proportion, dominance, movement, and economy [2], I can proceed through phase one of this mathematical methodology, which I call shape exercises. Whether working in two or three dimensions we find that line is found in nature either as a contour or a cross contour of some object, which has mass and/or volume. In two-dimensional art line can exist by itself as a mark on a piece of paper or it can be drawn back onto itself and create shape. Shapes are the fundamental building blocks of design. They are found everywhere we look in nature, science, and art. The history of Eastern as well as Western culture (via Euclidean Geometry) has imprinted upon our minds certain fundamental shapes from which all other shapes (at least in our human manner of description) are derived. When we make art we tend to idealize somewhat the shapes we use and we derive invented shapes from a combination of these idealized shapes. These shapes are basically the circle, the triangle, and the square (or rectangle). These are regular polygons and can be extended into other more complex polygons such as the pentagon, hexagon, octagon etc. Where did these shapes come from? They are and were fabrications or patterns of the mind. The Greek Philosopher and Mathematician, Euclid (circa 300BC) collected and organized many of the previously illustrated geometric ideas about shape and space that had been created since the time of his distant predecessor, Thales (640-546BC). This was an enormous task and eventually he compiled all of this information into a logical, mathematical system, which became known as Euclidean Geometry. He formally arranged his collection of information into a book, entitled, *The Elements*. Euclid was a purist. He insisted that the only way to create these regular shapes was from a most naive and fundamental method which did not depend on any a priori knowledge except that which was self evident such as the first five postulates of his geometry book, *The Elements* [3]. Those postulates are:

1. It is possible to draw a straight line from any to any point.
2. It is possible to produce a finite straight line continuously in a straight line.
3. It is possible to describe a circle with any center and radius.
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less that the two right angles.

As it turns out the fifth postulate was not so self-evident and eventually gave rise to what we call non-Euclidean Geometry. In any event, for Euclid and his followers and for our design students, the process of creating shapes or polygons was and is to be one of self discovery based on nothing but the five postulates as a starting point. To this end Euclid restricted himself and others, as we do our students, to the solitary use of compass and straight edge, the simplest and most immediate of tools, for the invention of regular polygons.

Because of the work of Euclid and subsequent geometers, we find the history of geometry labeled as either Euclidean or non-Euclidean. The preoccupation of the many aspects of shape in the history of geometry as well as in other aspects of the overall history of mathematics has lead contemporary mathematicians to more accurately describe mathematics itself as a science of patterns [4]. If we think about it, we could call design a science of patterns. At the very least, design is the mental fabrication of an idealistic picture of the world revealed through the presentation of patterns either two dimensionally or three dimensionally. And herein lies a major problem. Art (design) has often been called "an imitation of nature." But we know that nature does not reveal itself to us in regular shapes or polygons, as Euclid would have it. Nature is anything but regular or predictable. So we ask ourselves, "Where in nature do we find, for example, a perfect sphere, an equilateral triangle, or a cube?" No where! Appearing to be chaotic, nature seems to defy geometry, at least Euclidean geometry.

With this as background, students are asked to begin a journey of exploration into the ideal world of Euclid and create regular polygons (circle, triangle, square, pentagon, hexagon and octagon) with only compass and straight edge. Furthermore, students are asked to write the algorithm for creation of each polygon and be able to visually prove its veracity before the other members of the class. The most difficult polygon to achieve with only compass and straight edge is the pentagon. Perhaps one or two out of every twenty students is able to achieve this five-sided polygon without outside help. The student response to this first phase of retracing history and rediscovering the origins of the shape has been overwhelming and positive.

Phase two of this journey of mathematical exploration into the elements and principles of design consists of what I call spatial exercises. In order to work successfully in the third dimension we must fine-tune our perception of that dimension. In art we have traditionally started our investigations of images on a two dimensional surface with height and width as descriptors. As our personal and cultural experience broadens and we become more visually sophisticated, we begin to create illusions of the third dimension on the two dimensional surface by attempting to depict the dimension of depth. This illusion is made believable to ourselves and our viewers by the joining of simple two dimensional shapes according to the laws of linear perspective which were first explored over 600 year ago by such artists as Da Vinci and Durer. Though the fundamental ideas of perspective were discovered in the fifteenth century, and gradually came to pervade the world of two-dimensional art, it was not until the eighteenth century that projective geometry was studied as a mathematical discipline. If Euclidean geometry corresponds to our mental conception of the world around us, projective geometry captures some of the patterns that enable us to see the world the way we do two-dimensional images on our retinas. The basic idea of projective geometry is to study those figures or patterns, and those properties of figures or patterns that are left unchanged by projection [5].

The next step in phase two is to make the transformation from the illusionary two-dimensional world to the reality of the three-dimensional world. Students do this by taking a shape or polyhedron such as a cube or pyramid from a drawing and then making it into an actual three dimensional reality. This task is accomplished by mentally dissecting the rendered cube, or pyramids etc. and flattening out the shape into a pattern or net, as it is called. After adding tabs and flaps the net is cut out and assembled into a totally, enclosed, three-dimensional Euclidean object or module.

Now that we have made the transition from the ideal two-dimensional world of Euclidean shapes or regular polygons and successfully created three dimensional renditions or polyhedra, we are ready to come face to face with the real world of nature and see where Euclid's idealization has failed us. One reason lies in the fact that Euclidean geometry cannot describe the shape of a cloud, a mountain, a coastline, or a tree. "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lighting travel in a straight line" [6]. More generally, we can see that many of the patterns of nature are so irregular and fragmented that they cannot conform to Euclid's vision of the world at all. Nature truly exhibits an altogether different level of complexity. The number of distinct scales of length and dimension of natural patterns is for all practical purposes infinite. The existence of these patterns challenges us to study those forms or shapes that Euclid leaves aside as "being formless," to investigate the morphology of the "amorphous." For several hundred years mathematicians have disdained this challenge and have increasingly chosen to flee from nature by devising theories unrelated to anything we can see or feel. Twentieth century artists, on the other hand, have attempted to deal with these patterns of nature through abstract art, but until now have lacked an adequate scientific vocabulary. Responding to this challenge, from a scientific and mathematical perspective, the contemporary mathematician, Benoit Mandelbrot (b.1924), conceived and developed a new geometry of nature and implemented its use in a number of diverse fields including art. This geometry describes many of the irregular and fragmented patterns around us, and leads to full-fledged theories, by identifying a family of shapes called fractals [7].

Fractals are qualitative; a measure of the relative degree of complexity of an object as opposed to quantitative as in the measuring of length or width. Infinite detail, infinite length, and fractional dimension characterize fractals. Fractals exhibit self-similarity and they can be produced by iteration (feedback involving the continual absorption or enfolding of what has come before). Fractal shapes of great complexity can be obtained merely by repeating a simple geometric transformation wherein small changes in parameters of that transformation can provoke global changes. That is to say, when a random variation in mathematical iterations is allowed so that details vary from scale to scale, it is possible to mimic the actual forms and structures of nature much more closely. Fundamental to the study of fractals is the concept of self-similarity, which means a repetition of detail at descending scales. How could something that measures thousands of light years across have anything in common with objects that can be hand held? Could it be those similar mathematical laws and principles of growth are operating at such different scales? If this is true, Mandelbrot realized, then these laws must have little to do with classical Euclidean geometry, where scale is a notion so obvious that it is of little or no importance. Could one create a measure of irregularity that was based on scales? Scaling here implies that nature's shapes can exhibit irregularities and/or fragmentation that is identical at all scales. A simple and ordinary example of fractals wherein self-similarity and scaling are at work would be a living tree where branches have smaller branches (bifurcation) with details being repeated down to the dimension of tiny twigs [8].

Mandelbrot's central concept, which gives his book its title, *The Fractal Geometry of Nature*, is that the notion of three simple dimensions is a myth. Real-world objects occupy a space whose dimensions are fractional or fractal (from fractua- irregular and frangere- to break into irregular fragments). One kind of less than three-dimensional object may have 1.25 or 2.65 dimensions more or less etc. Dimensions of one, two, and three are theoretically possible but abnormal. (Example: sheet of paper-2D, crumpled to a ball-3D, unfolded but wrinkled-2.5D)

With the above as an introduction, phase three of my mathematical approach to design, invites the students to develop an original module, irregular (fractal) or if not, regular, and create a modular construction that exhibits self-similarity and scaling through a process of iteration. This approach has been well received from our students and has produced unusual and stimulating results on their part. Specifically, the objectives of phase three are:

1) To examine at least three, three-dimensional forms in the world outside the mind and attempt to identify their fractal nature.
2) To develop a single module that exhibits fractal characteristics.
3) To construct one or more complex aggregates from the iteration of the single module that exhibits fractionalization, self-similarity, and scaling.

During the balance of this mathematical approach to a three-dimensional design course of study, additional phases based on mathematical concepts are used to explore tectonic and atectonic shape/planar constructions as well as linear constructions of various mathematical configurations. All students in the course are limited to the use of specific materials (which are predetermined) in the execution of their various projects and are required to discuss the mathematical origin of their constructions. The overall student response to this mathematical approach to the design process has been positive and rewarding.

Historically, art students have avoided any contact with mathematics and the sciences; however, since our university has enacted a common core curriculum for all students, art students are now required to take mathematics and science as part of their general education. This fact has reinforced the use of mathematics in our art school and likewise has helped our students overcome their hesitation about mathematics and the sciences altogether, thus producing a more-well rounded artist.

References

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