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Mathematical Connections
in Art, Music, and Science

Problems with Holbein's Ambassadors and the Anamorphosis of the Skull

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Abstract

Holbein's *The Ambassadors* at the National Gallery in London has recently been restored with a great deal of criticism. Much of this has been centred on understanding the anamorphosis of the skull. The publications of the Gallery on the subject do not inspire confidence and the restoration has probably destroyed evidence which may never allow the nature of its construction to be determined. Science has become part of art restoration; the gross errors made show that mathematicians and art historians need to be working together also. It shows why artists need to be taught to think using the scientific method, which has been frowned upon in UK schools. There is an urgent need for bridges to be built between disciplines.

Introduction

Hans Holbein's *The Ambassadors* shown in Figure 1 was painted in 1533 and is one of the most famous

Figure 1

paintings in the National Gallery in London. Apart from its artistic, historical and symbolic significance, it has much scientific and mathematical content. Measuring instruments such as sundials, a torquetum, globes (terrestrial and astronomical) are displayed together with standard geometrical instruments such as dividers as well as a copy of Peter Apian's book on arithmetic of 1527, open at a page on division. There is more background to the painting in Appendix 1.

One of the most interesting objects in the painting is a skull inclined at an angle of about 24° (perhaps at the angle of the ecliptic) which is elongated in such as way as to require special viewing in order to be seen correctly. Such an image is known as anamorphic, a word derived from the Greek from the Greek *ana* 'back' indicating a return towards and *morphe* `form' [1]. The image is constructed so that it is distorted and needs to be viewed from a certain point or in a certain way to return back to the normal form. Since such images are meant to be viewed, they must be optical transformations. The skull is there as a reminder of death, a *memento mori.*

Some images also require mirrors or lenses to be seen correctly, but usually anamorphosis is a form of perspective. All exact perspective requires that you look from one (and only one) particular viewpoint to recreate the image as the artist saw it. With anamorphic images this viewpoint is so special that the image is extremely distorted when viewed from anywhere else. In the case of the skull there are two main theories as to the way the anamorphosis was created which I will discuss later in detail. The first is a special type of perspective (which I believe is the most probable), the other is that it required a cylindrical lens. The National Gallery also came up with another method which is against the laws of optics. It illustrates a classic case as to why art historians and restorers need to consult with external experts, and holds other important lessons in the way misinformation can be disseminated.

The Micro Gallery and Microsoft Art Gallery

The National Gallery was one of the first galleries to display their collection electronically with a set of touch screens which they called the Micro Gallery and which was subsequently sold as Microsoft's Art Gallery CD-ROM in 1993. Amongst other errors, the software has an animated "reconstruction" of the anamorphosis of the skull in The Ambassadors which is pure fantasy. I had much correspondence with the curator who acknowledged the error, but over three years, nothing was done to correct this misinformation even when they also admitted it in the press. The error continues to be perpetuated through the Microsoft CD and so may be spread like a virus.

Before describing the errors in detail, it is necessary to understand the hypotheses of Holbein's construction and viewing of the anamorphosis and how they affect the hanging of the picture.

Basic perspective

In describing these hypotheses, I am assuming a basic knowledge of perspective since I do not have space to go into it here, but a few points are worth bearing in mind. A perspective picture is formed when rays from an eye point are drawn to points of the object being drawn and where these rays intersect the picture plane, the image point is created. Straight lines in space remain straight lines in the picture. The spacing of the lines changes according to a non linear law. Finding this relationship in the fifteenth century was the key to drawing in perspective. It was achieved by drawing perspective grids like the one in Figure 2.

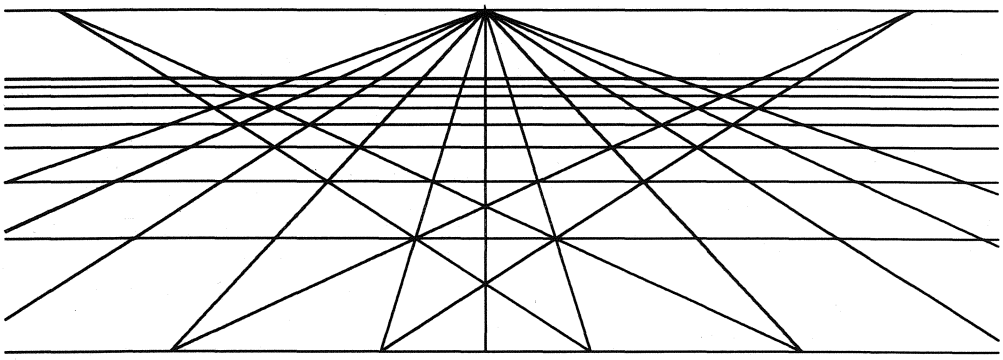

Figure 2

The picture plane is normally perpendicular to the line of sight. Where it is not, and the picture is then viewed perpendicular to the plane, the image appears distorted. This is an anamorphic image.

Hypotheses

Anamorphosis was first studied by Leonardo da Vinci as an extreme form of perspective where the picture plane is viewed at oblique angle. It was studied in its various forms by many artists and mathematicians both as an illusion and for scientific reasons in the study of optics. Methods for construction appear in books on perspective and in books on mathematics. One of the most famous is that by Niceron who was a pupil of the mathematician Mersenne [2]. One of his illustrations for copying a picture using a grid is shown in Figure 3.

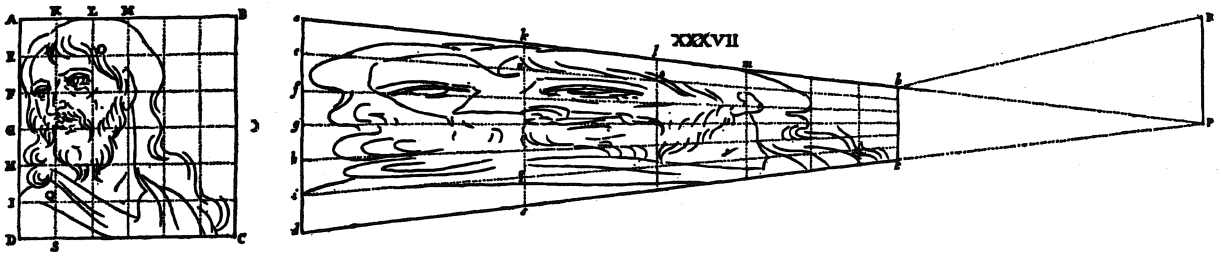

Figure 3

To view the image correctly, place your eye above the paper at the point P at a distance of RP. Looking from the left does not restore the correct image. Note how the square becomes a trapezium which tapers close to the viewpoint and has the most distortion at the side opposite to the viewing one.

Very simple methods were used which are not strict perspective, which just elongate the image. They show no tapering and so are easily identified. For example, Baltrusaitis [1 page 35] shows a picture of a dog which uses what he calls "the old-fashioned anamorphic method". The original has a square grid and the anamorphic version simply stretches the grid in the horizontal direction by a factor of three. The picture shown is by Samuel Marolois and is dated 1614. The method seems to have originated in Vignola [5].

Now look at the skull in the Ambassadors (Figure 4), bearing in mind this tapering and that there is only one point from which to view in a correct perspective. The conclusion is that it should be viewed from the right. This is the first hypothesis.

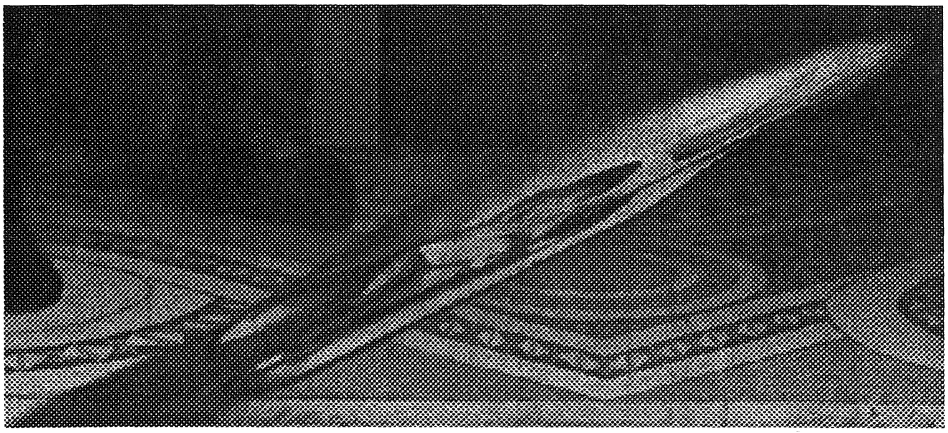

Figure 4

However, this is coming from a position where we know a great deal about viewing pictures in perspective and so we may be prejudiced by this. The skull is not a sphere and viewed in this way has more substance around the jaw bone than that at the base of the back and this may be giving rise to some of the tapering. But the relative sizes of the eye sockets would not occur in a simple elongation, so the evidence for the first hypothesis that the image should be viewed from the right to restore it back to the one we would expect is very probable. This hypothesis has been the accepted one ever since the painting went on display in the National Gallery about a hundred years ago.

A second hypothesis was put forward by an English optician, Edgar Samuel in 1963 [3]. He suggested that the image might be restored by viewing it through a cylindrical lens, that is a glass tube (not a rod of glass which would cause a left right inversion of the image), held perpendicular to the direction of the skull. The tube would point to the face of De Dinteville at the left and the skull appears as a roughly spherical object forming a series of such objects (with the terrestrial and celestial globes) in a vertical line. This method requires a special device and this could have added to the mystery of a painting already strongly full of symbolism. Using such a tube gives a plausible image. I feel that it is improbable that Holbein used this method for a number of reasons. The image is very small and the method does not occur in any of the many books (right up to the present day) demonstrating anamorphosis, since optics based on lenses is a relatively late development. (See appendix 2 for more details). Moreover, taking an image known to be in perspective, like the Niceron one, or the William Scrots portrait of Edvard VI in the National Portrait Gallery in London [see 1] yields an acceptible image with such a lens. I agree with Samuel [3] that it would be impracticable to paint while viewing through a lens and that some form of copying grid would be required.

There are other puzzles of perspective in the painting. Holbein has a good eye yet he cannot draw the image of the circles on the floor accurately. The one on the left (which holds De Dinteville's foot) is a very crude representation of the ellipse expected from perspective. Perhaps he used an assistant who was not very skilled. The grid method used by Niceron shown above is hard enough to create. It is certain that Holbein would make a grid for the lens method if he used it. It may even be that Holbein used someone else to create the grid for the perspective, perhaps Nicolas Kratzer who was Henry VIII's instrument maker and a friend of Holbein.

Optically both hypotheses are plausible, and the human brain is ideally equipped to understand the images restored by either method. Ultimately, this is the only test we can apply. Imperfect painting or erroneous restorations do not deter from our ability to recognise it as a skull in either case, but they do muddy the waters for a scholarly need to know what was the actual method used. Moreover, such mathematical analysis needs to be used before restoration in the same way that chemical analysis is undertaken on the pigments and infra-red and X-ray photography to determine surface and underlying structure to the

painting. Unfortunately, the National Gallery have difficulty understanding the perspective and the recent restoration may have lost what evidence there was for ever.

The Micro Gallery Error

There were two errors in the theory put forward by the National Gallery. First they had the picture hung on a stair, suggesting that the user would see the image from two possible foreshortening points - one of which would be clearly have to be wrong if the skull was distorted using perspective and possibly both wrong if some other method of distortion was used in painting it.Their second error, used in the animation of a reconstruction was to take an image of the skull, superimpose a grid on it (of regular spacing) and then shear the image so that the spacings remain equal. The before and after effects are shown in Figure 5. Apart from the restored image not looking correct, it would be impossible for the spacing to remain the same. My guess is that they have given the problem to the computer artists who are ignorant of perspective and they have come up with their own solution which has not been checked by someone who understands the properties of perspective. As I describe below, computer programmers creating graphics software can produce some unexpected effects which do not match the real world.

The resulting sheared image at the right of Figure 5 is almost correct (except that the left eye socket is too big), although I suspect that this may not be the full story and the restored image may have been produced in a another way.

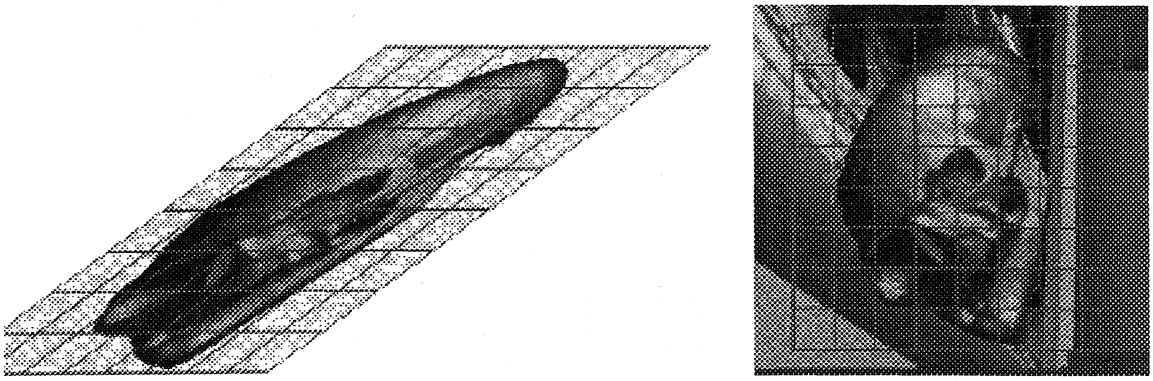

Figure 5

Errors in the Catalogue

After the current restoration, The Ambassadors was put on display in 1996. It was hung in a place in the gallery, where it was impossible to look at the skull to restore it! At the end of 1997 there was a special exhibition on the painting with a catalogue [4] which attempted to explain how the skull had been restored. It discusses both the theories outlined above. There is not room here for a complete criticism of the explanation presented. There is a complete doctoral thesis awaiting someone with an understanding of perspective to work on this aspect of the picture. It is a classic instance of why such a gallery needs the help of scientists and mathematicians as part of the restoration team and more so to communicate what they have done. I will only concentrate on the most glaring errors.

They describe how the standard perspective (the first hypothesis above) is feasible. The correct qualitative result is given, although this is described in a somewhat confusing fashion. Grids have been drawn with the

correct geometry, although there is no attempt to show the image on the grids as in the Niceron example (Figure 3). There is no indication as to how a regular rectangular (not square) grid was distorted to a trapezoidal one is achieved. They say,"The resulting image is convincingly resolved". It does not convince me, because the "starting" image has already been distorted from the original and is half way to being restored, with no indication (or acknowledgement) of how this might have occurred. It all looks very much like adjusting the results to fit the question.

This hypothesis was used to repaint the skull during the restoration. One problem was that the nose area had been previously restored and the cleaning removed all paint from that area. In order to determine how to recreate this, the computer was used, but unfortunately with insufficient thought. A complete skull was photographed and distorted by computer. (Reservations about this are described below.) In the original painting the person whose skull was used was probably quite typical of the time: it had few if any teeth left. The restorers used a modern skull, one with complete dentition. From a scientific viewpoint this is not a valid route to take. From an artistic point of view it is disastrous. The teeth have placed the jawbone in a different place and this has resulted in a skull which is neither the original nor the new one. Restorers do not normally paint outside the limits of the old paint. In this case the jawbone protrudes into the border which it did not do before.

It is with the second hypothesis that scientific method goes out of the window. The scientific approach would be to look through a glass tube and see if the hypothesis is valid, as indeed it is, and then to investigate further. The description adopted to disprove the hypothesis is enough to make even the most tolerant scientist's or mathematician's heart sink. The light is "reflected" (sic) in the lens, but even so "an excellent image of the undistorted skull may be seen in this way." Then the computer is used in a clumsy way. The method described is that a grid is used again, but unfortunately the one shown has the wrong spacing. Instead of the spacing being narrow in the centre and wide on the outside (so that maximum compression occurs at the outside, as it will with a cylindrical lens) it has wider spacing at the centre. This error is then used to produce an image which is distorted in the wrong way. Instead of being restored, the distortion (Figure 6) is increased!

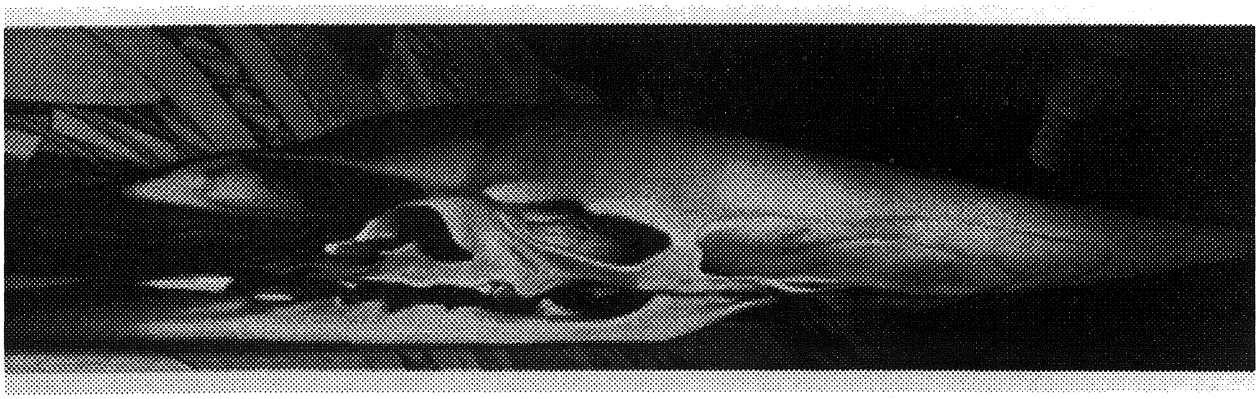

Figure 6

Because the computer could not be made to create the correct image, the National Gallery then say that this method could not have been used. Can there be a more illogical result? How can they ignore the images in their eyes, and not deduce that there is something wrong with their transformation?

Note

These explanations are in a catalogue which has been written for a non-technical audience. The National Gallery Technical Bulletin (Volume 19) describes the restoration in slightly more detail but, sadly, provides no more facts.

The computer and perspective

Bridges also need to be made to programmers creating graphics software. It is very common to find a "perspective" option in such software. Unfortunately, I have yet to find one that obeys the mathematics and optics of perspective.

This can be tested by drawing a square grid and some diagonals and then performing the perspective transformation using the software. The following examples show how straight lines become curved and how the picture is distorted. It is always worth performing such tests before using this feature if it is important.

The first case is the vector based software Corel Draw. The grid on the left in Figure 7 has been altered using the perspective effect to give the Corel version on the right. Note how the diagonals have become curved instead of remaining straight. The plane consequently appears curved not flat.

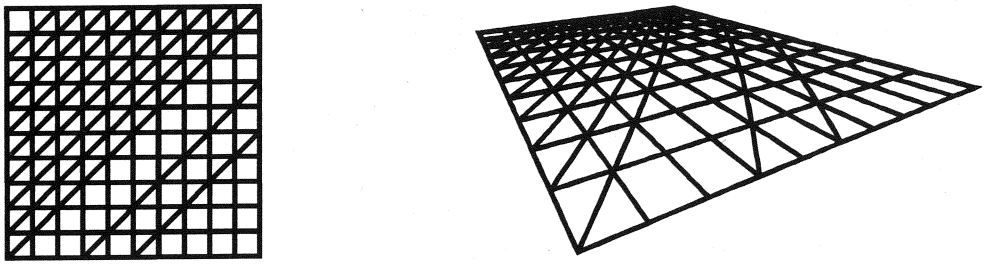

Figure 7

The second case is the popular PaintShop Pro which works on pixel graphics. This has the most severe problems. Not only does the perspective give diagonals which are markedly curved, but the foreshortening is wrong. If you imagine this as fence posts, you would expect that the spacing would get closer as you go into the distance. In fact, you get a spacing which gets wider!

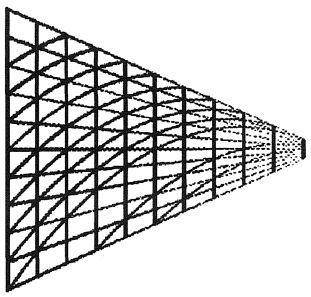

Figure 8

There is a possibility here for the exploration of non-euclidean mathematics and to get some interesting graphical effects both for conventional images and abstract ones. The tessellations which can be created also offer new possibilities for areas such as quilting and patchwork.

Conclusions

The Renaissance re-discovery of perspective in the early fifteenth century was made at a time when a broad education included science mathematics under the umbrella of the seven liberal arts symbolically depicted in the painting. Artists like Durer and Piero della Francesca, particularly, were mathematicans in their own right and wrote mathematical books. We are now swamped with knowledge and information, so that it is impossible for even a mathematician to understand all mathematics.

In studying art history, especially where restoration of unique paintings is involved, it is vital that teams are drawn from all disciplines, bridging the arts sciences and mathematics. Art colleges need to teach basic science, particularly optics. The lessons of the National Gallery's treatment of The Ambassadors holds many lessons for future restorations. Whether they will be learnt is another matter.

Appendix 1 - Background to the Ambassadors

The painting is much travelled. It was probably painted in England when De Dinteville and De Selve were ambassadors to the court of Henry VIII at the crucial time when the King was haggling with Rome over his divorce from Catherine of Aragon. Easter 1533 was the deadline he had set for an answer. France had acted as a go-between and so the French Ambassadors were very important visitors to his court at this time. Henry's patience ran out and the Church of England broke from Rome with consequences for both the history of England and the balance of power in Europe. De Dinteville took it back to France and it returned to England in the late eighteenth century during the revolutionary upheavals in France, to be purchased by the Earl of Radnor in 1808. It was sold to the National Gallery in London to offset death duties and so came into public ownership in 1890.

The size of the painting is 207 cm by 209.5 cm (81.5in by 82.5in) so it is very impressive and the figures are life size.

Following the recent restoration, there was an exhibition putting the painting in context both historically and together with other work by Holbein in late 1997 and early 1998. This exhibition allowed visitors to look from the right side to see the skull in its restored state. Now it is back on display as normal, there is a rail around it so that you can no longer reach this position. The exhibition also had a reproduction of the painting which had a glass tube in front of it to allow reconstruction by Samuel's method.

Because of the wide length of the skull, it is difficult to restore the image photographically since you need a long depth of field, but viewing it with the eye is much easier. The brain scans the image and you focus on different parts of it. If it were in a room with a door to the right, if you glanced back as you left the through the door, you would easily see the image restored. The viewing point is consistent with the height of a person like De Dinteville if you allow the painting to show him as life size.

The history of the painting and of the subjects is described in Hervey [8]. It is summarised in Baltrusaitis [1]. A symbolic interpretation is given in [7].

Appendix 2 - Sources of anamorphic art methods

The reference [1] above is one of a series of editions of the book by Baltrusaitis, the latest of which is only in the original French and was published in 1984. It is an art historical survey and there is no attempt to

look at the mathematics of the constructions which seems hardly to have been researched in this century. It has excellent, extremely comprehensive, references. The two main sources of illustrations and practical methods are perspective treatises and books on scientific and mathematical recreations, such as those by Ozanam and Hutton. Niceron [2] mentioned above is the first book devoted to the subject. Other perspective books such as Vignola's [5] and perspective books such as Du Breuil's which went through several editions after 1650 contain sections on such perspective tricks.

There are also references to machines for creating anamorphic images, such as Jacob Leupold's *Anamorphia mechanica nova* published in Leipzig in 1713.

The main methods for creating images are flat perspective ones from unusual viewpoints (as shown in Figure 3), images on cones and polyhedra and on zigzag surfaces which need to be viewed from two different directions. Then there are images to be viewed using polyhedral, conical or cylindrical mirrors.

In slowly working through original references, I have only come across one type of image that uses refraction of the light rays forming the image. It uses not a lens, but a glass prism with multiple facets which bring together parts of the original image to form another one. These methods are passed on from book to book. If a lens method as proposed by Samuel had been used at any time, it would have surely proliferated.

References by modern art historians are not always reliable, and it is necesssary to go back to original sources. I mentioned that the National Gallery catalogue talks about "relection" through a lens. It also describe an inventory of pictures which mentions a "Sellinder glass" as part support of the Samuel hypothesis, converting the "glass" into lens. On looking up the original reference, it was obvious it was referring to a cylindrical mirror, because of the description of where you placed it as well as other references to mirrors. When interpreting old text one must be aware that the term glass was used in the way that a mirror was called a "looking glass" as in the title of Lewis Carroll's book "Alice through the looking glass". Another example where I thought I had found such a reference to lenses is in the book [6] by Barbara Maria Stafford who is Professor of Art History at the University of Chicago. Following her reference to Leupold mentioned above, I expected to see how "By a crafty sleight-of-hand, cylindrical and conical lenses were made to rectify deformed images...", but only found references to mirrors. Following historical references is far easier to do in Europe where originals exist. It also requires a wide knowledge of different disciplines, languages and cultures. Perhaps there also need to be bridges across the Atlantic in joint research.

References

[1] J Baltrusaitis, *Anamorphic Art,* Chadwyck-Healey Cambridge 1977, a transation of the French edition

[2] J-F Niceron, *La Perspective curieuse,* Paris 1638

[3] Edgar Samuel, *Death in the Glass*, Burlington Magazine, 1963, pp 436-441

[4] Susan Foister, Ashok Roy and Martin Wyld, *Making and Meaning - Holbein's Ambassadors,* London, National Gallery Publications 1997 (ISBN 1-85709-173-6)

[5] G Barozzi da Vignola, *Le due regole della prospettiva pratica*, Rome 1583

[6] Barbara Maria Stafford *Artful Science: Enlightenment, entertainment and the eclipse of visual education* MIT Press 1994

[7] Richard Foster and Pamela Tudor Craig's *The Secret Lives of Paintings* London 1986.

[8] Mary Hervey *Holbein's Ambassadors, the Picture and the Men*, London 1900